THE RIVER KLARÄLVEN

A STUDY OF FLUVIAL PROCESSES

BY

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PREFACE

The object of this work has been on the one hand to present a synopsis of recent facts and theories regarding fluvial processes, including erosion, transportation and deposition of sediment, and on the other to apply some of these results to investigation of an actual river. Erosion of rock—corrasion, evorsion, and cavitation—have not been considered.

The first chapter deals with the mechanics of flowing water, and is a concise account of flow phenomena that are of fundamental significance for fluvial processes. The second chapter contains a presentation of various aspects of fluvial processes dealt with in the literature, together with the author's own treatment of these processes and of certain morphological and stratigraphical problems. In the third chapter there is a description of some new methods for field measurements. The fourth, and final, chapter is devoted to an account of the fluvial morphology of the upper Klarälven.

This work was made possible by invaluable assistance from many private persons and institutions, to whom I bear a great debt of gratitude.

Professor Filip Hjulström gave the initial impulse to this study, and has subsequently followed it with the greatest interest through all its phases. His outstanding knowledge on problems of fluvial morphology and his own pioneer investigations in this field have been my best support during the course of the work. He has also read through the entire manuscript and made many valuable comments.

Several of my older and younger colleagues at the Geographical Department of Uppsala University have in various ways made important contributions to the completion of the work. In many cases it has entailed a very considerable personal sacrifice of time and energy. Among those who have helped me I would especially like to mention Professor Gerd Enequist, who has read through the manuscript of the section Changes in the shore lines; Docent Sten Rudberg, who has read the greater part of the proof with attention to both factual and formal aspects; Lennart Arnborg, F.L., with whom I have discussed particular problems, and whose wide field experience has been of great help; Valter Axelsson, F.M., who has given much good advice during the course of the work, and who has read the greater part of the manuscript; Åke Falk, F.M., who has investigated important morphological details in the valley of Klarälven, and who has placed the results at my disposal; Sölve Göransson, F.M., who has read the proof of the section Changes in the shore lines; Börje Möller, F.M., who has allowed me to use some results of his soundings in Öre älv.

Soil analyses, treatment of statistical data, and various computations and excerpts

have been carefully carried out by H. Gauja and Helgi Köpp. The final copies of maps and diagrams have been skillfully drawn by Capt. W. Tiit and Emma Balodis.

Since 1953 the investigations in the valley of Klarälven have been carried out in collaboration with the Swedish Geological Survey. This has been a great advantage to me, both scientifically and practically. I am therefore greatly indebted to Professor Nils H. Magnusson, the Director of the Survey, Docent Carl Caldenius, former State Geologist, on whose initiative the collaboration began, and Jan Lundqvist, F.L., who has led the work of the Geological Survey in the valley of Klarälven during recent years.

Professor Bo Hellström kindly allowed me to carry out certain experiments at the Hydraulic Laboratory of the Royal Institute of Technology in Stockholm. He has also given me information regarding literature otherwise difficult of access, and even in other ways shown his interest for the work in progress.

On several occasions I have been able to borrow instruments for hydrological measurements from the Swedish Institute of Meteorology and Hydrology. The helpfulness of Dr Ragnar Melin, Head of the Hydrological Department and Dr Olof Tryselius, State Hydrologist, in this respect facilitated the field work to a considerable degree.

One of the important impulses for this work originated from a visit to the Norwegian Vassdragsvesen in Oslo, where I had the opportunity to study the Norwegian methods for measuring the discharge of a river by means of the salt-dilution method. Mr R. Sögnen, Chief Engineer there, helped in all ways to make my stay in Oslo as fruitful as possible. Mr S. Nordnes, cand. real. and Hydrologist, introduced me to Mr Sögnen, and he has also been very helpful in other ways.

It has been a great satisfaction to me, a native of Värmland, that the greater part of the field work took place in that province. The work there in the valley of Klarälven has been extremely stimulating, not least on account of good contact with local people. Among representatives of the local authorities who have assisted me in various ways, I would especially like to mention Mr A. Svanhult, and Mr G. Löfgren. The assistance given by Mr S. Lundin in making soundings, taking water samples, and in other field work, has been particularly valuable.

During the work in the valley of Klarälven it has often been necessary to contact representatives of Uddeholms AB and Klarälvens Flottningsförening. From them I have always received unhesitatingly all the help I have wished for, whether it has been a matter of accomodation, organisation of sampling, etc., or even direct financial assistance. Foremost among the representatives for Uddeholms AB were Mr J. Fletcher, Managing Director, Mr J.-E. Jarlås, and Mr G. Sandling. Mr B. Lundén, Director in Klarälvens Flottningsförening, has also been very helpful.

The translation from Swedish is due to Mr R. Stedman, who has also read the first proof.

Finally, I would like to mention the help and encouragement I have received from members of my family. My wife, Gerd Sundborg, F.M., has taken an active part in the entire work, particularly in the completion of the erosion map and during the printing of this volume. My parents, Fritjof and Ester Sundborg, have followed the work with continual interest and have in every way sought to facilitate it.

Financial contributions for the field work have been received from the Andréefond, Liljewalch's Stipendiefond, Lennander's Stipendiefond, the Field Research Fund of the Section for Mathematics and Natural Science, Uddeholms AB, and Klarälvens Flottningsförening; for the translation from the Längman Kulturfond, and for publication from the State Council for Scientific Research.

To all the persons and institutions—even those not mentioned by name here—who have helped in the completion of this work, I wish to express my sincere thanks.

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ÅKE SUNDBORG

CONTENTS

CHAPTER I

THE DYNAMICS OF FLOWING WATER

General

The bed of a river is subject to continual changes from the action of the flowing water. The type of changes, and the rate at which they proceed, depend on the manner in which the water flows and on the properties of the river bed. The state of flow determines the active forces, while the geological condition of the river bed determines its reaction to these forces. Fluviatile morphology, and in particular the study of fluviatile processes, must therefore be based on results from other branches of geological and physical research. The subject of fluviatile morphology is a striking instance of an intimate coordination of adjacent scientific fields, predominately theoretical hydrodynamics, practical hydraulics, physical geography, geology, theoretical and practical soil mechanics, and the wide branch of soil science.

Modern investigations of fluviatile processes, e.g. erosion, transport, and sedimentation have generally adopted as initial working hypothesis some conception of fluid flow, based either on experiment or on theory. The morphologically important processes have been treated from the viewpoint of this hypothesis (cf. MATTHES 1947, KALINSKE 1945 and 1947, and EINSTEIN 1950). The present paper follows a similar presentation in the main.

It is outside the scope of this work to take up a detailed treatment of the dynamics of flowing water. Various standard works in hydrodynamics and hydraulics (FORCHHEIMER 1930, GOLDSTEIN 1938, PRANDTL 1952, ROUSE 1950, and RICHARDSON 1950) deal thoroughly with the subject. Nevertheless, for the sake of continuity in the reasoning, the following section gives a brief account of those flow phenomena which are of primary importance for morphological processes, and which should be understood in order to follow the subsequent presentation.

According to the manner in which the particles of a flowing fluid move, the flow may be assigned to one of two types, either laminar flow or turbulent flow.

Laminar flow (or streamline flow) is characterised by the condition that layers of fluid slip over contiguous layers without mixing between the layers, apart from a negligible molecular mixing. In laminar flow the stream lines remain separate from one another. Laminar flow may be constant or variable with respect to space and time. Flow which is constant in both speed and direction along the stream lines is termed *uniform*, while, if it is variable it is termed *non-uniform*. If the flow is at every point constant with respect to time it is called *steady*. If it varies with time it is *unsteady*.

Since all real fluids possess a certain viscosity there is an internal friction that resists flow. When a layer of fluid slides over another the friction between them gives rise to a *shearing stress*

$$\tau = \mu \frac{du}{dz} \tag{1}$$

where τ is the tangential force per unit area, μ the *coefficient of viscosity*, and $\frac{du}{dz}$ the velocity gradient perpendicular to the direction of motion. The shearing stress τ is tangential to the stream lines. The element of area considered is perpendicular to the z direction, and thus in the same plane as τ.

When the velocity gradient becomes higher, or under the influence of external disturbances, laminar flow is transformed to an eddying motion where there is intense lateral mixing. This motion is called *turbulent flow* (or *sinuous flow*). It is always unsteady and non-uniform. Turbulent flow may be regarded as a motion where a complex secondary movement is superposed upon the primary motion of translation.

For turbulent flow the analogue of equation (1) may be written

$$\bar{\tau} = (\mu + \eta) \frac{d\bar{u}}{dz} \tag{2}$$

$\bar{\tau}$ is the average value of the shearing stress at any instant, and $\bar{u}$ the average flow velocity. η is the so-called *eddy viscosity*. The molecular viscosity μ is of course the same everywhere in the flowing fluid, but η varies in both time and space. Equation (2) was stated in a slightly different form by BOUSSINESQ (1877), and was extended by SCHMIDT (1925), among others. Instead of η SCHMIDT used the symbol A, which he called the "*Austauschkoeffizient*".

When the flow is fully turbulent A is much larger than μ, and (2) may then be written in the form

$$\bar{\tau} = A \frac{d\bar{u}}{dz} \tag{3}$$

Instead of SCHMIDT's "Austauschkoeffizient", A, publications in English usually employ the *eddy viscosity* or *momentum transfer coefficient*, ε, where $\varepsilon = A/\varrho$ (ϱ = the density of the liquid). The disadvantage of using (2) or (3) directly is that A and ε are not material constants but dependent on, among other things, the speed of flow (cf. p. 156).

Already at this stage it may be pointed out that in turbulent flow the inertial forces dominate over the frictional forces, while in laminar flow the frictional forces are predominant.

Turbulence plays a decisive part in the processes involved in flowing water's morphological action. The nature of turbulence is, however, so complicated that many problems associated with it still remain unsolved despite continual research since the end of the nineteenth century. Current theoretical accounts of flow phenomena with fully developed turbulence give only a generalised picture of the reality. Here, in order to facilitate an understanding of the importance of turbulence for fluviatile processes, an account will first be given of the causes of turbulence, followed by a qualitative description of some important flow phenomena.

Dynamical similarity. Reynolds' number

Under what conditions is flow laminar, and when does the transition to turbulent flow occur? One way of characterising a particular flow—with implications regarding the state of flow—was given by OSBORNE REYNOLDS as early as 1883 (REYNOLDS 1883). He showed that two streams of liquid that are bounded by geometrically similar surface configurations themselves exhibit geometric similarity, if certain conditions are fulfilled. The flow is similar in the two cases if

$$\frac{u_1 l_1}{\nu_1} = \frac{u_2 l_2}{\nu_2} = \text{const} \tag{4}$$

where u is a characteristic velocity (e.g. the mean velocity for a particular cross section, or the velocity at a particular point—corresponding points and surfaces being considered in the two cases), l a characteristic length (e.g. the diameter of a body immersed in the liquid, or the thickness of a layer of liquid), and ν the *kinematic viscosity*, i.e. the coefficient of viscosity μ divided by the density of the liquid ϱ ($\nu = \frac{\mu}{\varrho}$).

If condition (4) is fulfilled there is said to be *dynamical similarity* between the two cases. $\frac{ul}{\nu} = R$ is a dimensionless parameter, which is called *Reynolds' number* in honour of REYNOLDS, and is customarily denoted by R. The condition for dynamical similarity may be regarded as signifying that the forces of inertia and the forces of friction are in a certain proportion to one another in the two cases, since the forces of inertia can be shown to be proportional to $\frac{\varrho u^2}{l}$, the frictional forces to $\frac{\mu u}{l^2}$, and $\frac{\varrho u^2}{l} \Big/ \frac{\mu u}{l^2} = \frac{ul}{\nu}$.

When R is small the frictional forces predominate. The flow is then laminar. As R increases the inertial forces increase, and at a certain *critical velocity* turbulence sets in. The magnitude of R is therefore important in characterising the type of flow in a particular case.

The value of R at which the flow ceases to be laminar varies within certain limits, however, depending on the external conditions. The roughness of the boundary surfaces is of especial importance. A smooth surface may give rise to a higher value of R_{crit} than a rough one in the same position. A small disturbance may render a previously stable flow unstable, without any change in the value of R. There thus exists a region of transition where either type of flow may occur, depending on the prevailing external conditions. In some cases this interval may be very large: instances have been observed where turbulence occurs at values of R more than 10 times smaller than that in other experiments where the flow was laminar (SCHILLER 1922, p. 16). However, there does appear to be a lowest critical value of R below which the flow remains laminar whatever disturbances occur.

In an open channel or natural water-course ALLEN (1934) found R_{crit} to be 1,400, with u in equation (4) representing the mean velocity in a given cross section, and l the hydraulic radius, i.e. the area of the cross section divided by the wetted perimeter (the length of the cross section's perimeter that is covered by water). The hydraulic radius for a broad channel of uniform depth is nearly equal to the depth. Other investigations have given considerably lower values of R_{crit}, and the generally accepted value is c. 500 (cf. ROUSE 1950, p. 83).

It is open to discussion whether a critical Reynolds' number is under all conditions an appropriate criterion of the transition between laminar and turbulent flow. Several investigations have shown that when the water depth is one or a few millimetres there may be an irregular flow of apparently turbulent type for values much lower than 500 (cf. HORTON, LEACH, and VAN VLIET 1934, and HJULSTRÖM 1935, p. 242). According to HORTON, LEACH, and VAN VLIET, the flow velocity corresponding to the lower limit for turbulent flow is given by Horton's criterion: $u_{crit} = 0.021 \frac{\nu}{n^2 D^{2/3}}$, where n is a coefficient of roughness of the bottom, and D is the depth of the water. R_{crit} is then proportional to $D^{1/3}$ and inversely proportional to the square of the roughness coefficient.

However, the low values of R_{crit} for very small depths may be due to the circumstance that gravity waves easily develop where the bottom is steeply sloping and rough (ROBERTSON and ROUSE 1941, p. 170). These so-called roll waves "may produce a local state of turbulence even though the undisturbed flow is well below the critical Reynolds' number". Experiments (HJULSTRÖM 1935) have clearly established that turbulent mixing of the flowing water occurs on account of these gravity waves. It therefore seems justifiable to refer to the flow as turbulent. As is later pointed out in the section on the initiation of turbulence (p. 139), the local turbulence caused by external disturbances when R is lower than its critical value cannot persist; the flow soon becomes laminar again unless new disturbances occur. Accordingly, it seems correct to describe the state of flow as laminar whenever R is less than 500. Nevertheless, local turbulence is probably of great importance for the erosion due to very thin sheetfloods.

Gravity waves. The Froude number

The ratio of the flow velocity of water in a channel to the velocity of propagation of small gravity waves is, like Reynolds' number, characteristic of the state of flow. Whereas Reynolds' number indicates the effect of viscosity, this ratio indicates the effect of the gravitational acceleration. The velocity of propagation of the gravity waves is given by

$$C = \sqrt{gh} \tag{5}$$

where C is the velocity of propagation, g the acceleration due to gravity, and h the depth of the water.

If the flow velocity u is equal to C, a wave disturbance evidently cannot travel upstream. This occurs when $u/\sqrt{gh} = 1$. The ratio $u/\sqrt{gh}$ is known as the *Froude number*, F, and, like R, is a dimensionless parameter.

When $F < 1$ the flow is described as *tranquil* (or *streaming*, Ger. Strömen), when $F > 1$ as *shooting* (or *rapid*, Ger. Schiessen). When the flow is tranquil disturbances can be transmitted upstream, and obstacles of various kinds have a damming effect which raises the water level upstream from the obstacle. But in shooting flow the motion upstream from an obstacle is not affected, and the water level there is lower than it would be for tranquil flow. Irregularities near the shore of a water-course with shooting flow give rise to small stationary oblique waves, while stationary cross waves are set up behind obstacles in tranquil water.

Where the flow of water changes from tranquil to shooting, the water level falls evenly. When there is a transition from shooting to tranquil flow, on the other hand, the change is often abrupt, and a turbulent surface roller, a so-called *hydraulic jump*, forms. At a hydraulic jump some of the kinetic energy of the shooting flow is transformed to turbulent energy.

If the Froude number F is only slightly greater than 1 (greater than 1 and less than a number somewhat less than 2, according to ROUSE 1950, p. 72) the transition occurs without a definite jump. But a series of standing waves is formed. Such waves are characteristic of flow where the depth is just slightly greater than the critical.

At small depths Froude's criterion is not reliable, because the surface tension then affects the wave velocity. Gravity waves have a least velocity of 23.2 cm/sec, but this applies only to waves on water deeper than about 0.5 cm (ROBERTSON and ROUSE 1941, p. 170). For depths less than 0.5 cm the situation is rather uncertain. Basing their conclusion on experimental observations, ROBERTSON and ROUSE consider the gravitational forces to be completely dominant, which means that $F = 1$ corresponds to a boundary between tranquil and shooting flow even at very small depths. EKEVÄRN and NORRBY (HELLSTRÖM 1945, p. 53), on the other hand, have concluded that Froude's criterion is not reliable for depths less than 1 cm, and that the transition velocity at that depth is 30 cm/sec. It thus seems likely that for small depths there is a transition zone between the two types of flow, where both may occur, depending on the external conditions.

The regimes of flow

The previous section has mentioned four different states of flow, which are gathered together in the following table.

Laminar ($R < 500$—2,000)		Turbulent ($R > 500$—2,000)	
Tranquil ($F < 1$)	Shooting ($F > 1$)	Tranquil ($F < 1$)	Shooting ($F > 1$)

The boundaries are indicated in fig. 1. For natural channels the boundary between laminar and turbulent flow is probably very near $R = 500$, because the great number of disturbing elements induces the transition to turbulence if the velocity is sufficient for

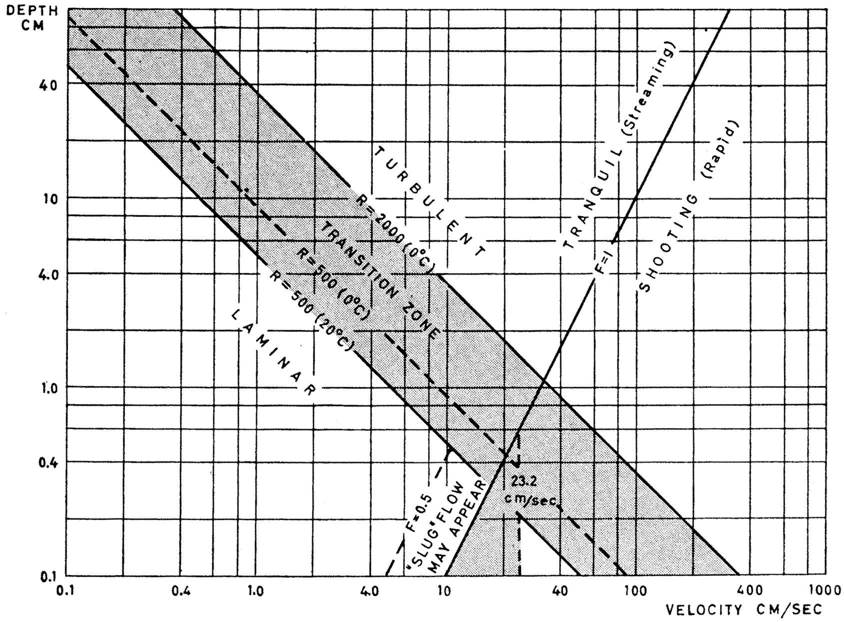

Fig. 1. The regimes of flow in a broad, open channel. Note the influence of the water temperature on the limit between turbulent and laminar flow.

permanent turbulence to occur at all. In laminar, tranquil flow over a rough surface there is often an intermittent uneven motion (intermittent or "slug" flow), with pulsation or undulation that causes local turbulence. This occurs when $F > 0.5$ (ROBERTSON and ROUSE 1941, p. 170).

It will be seen from the diagram that laminar flow occurs only for slow currents or for extremely small depths. In all natural water-courses the flow is turbulent, unless the water is practically stationary or moves in very thin layers (thin sheet flow). In so-called sheet-flow erosion the flow may evidently be laminar, either tranquil or shooting, though local turbulence is common and often dominating on account of the unevenness of the bottom.

The only exception to the dominance of turbulent flow in rivers and streams is in the proximity of the bottom. The flow there is under certain conditions laminar nearest the bottom, and there is then said to be a *laminar sublayer*. The conditions in the boundary layer will be considered in the next section.

Laminar and turbulent boundary layers. The initiation of turbulence.

The flow conditions in the contact zone between the flowing water and the river bottom are of very great significance for morphological processes. Some important phenomena associated with this contact zone will be dealt with below.

The water in *immediate* contact with a rigid body adheres to it, and is therefore stationary relative to the surface. If the body is immersed in flowing water the velocity of flow increases from the surface outwards, at first very rapidly, then more slowly, until the velocity of the freely flowing water is attained at a certain distance from the surface. The zone in which the flow is appreciably retarded by the viscosity of the water and friction against the surface is known as the *boundary layer* (Grenzschicht, Reibungsschicht). The theory for flow in the boundary layer was first treated by PRANDTL (1904), and subsequently has become one of the important fields of hydrodynamics (cf. SCHLICHTING 1951).

The flow in the boundary layer may be either laminar or turbulent. In channels or natural water-courses, where the boundary layer arising from friction against the bottom extends practically throughout the entire depth of the stream, the flow is of course generally turbulent (cf. p. 138). However, in order to understand how turbulence develops, the process by which a laminar boundary layer is transformed to a turbulent layer is of great significance, and we may therefore briefly consider it here.

The modern view of the initiation of turbulence is based on the idea that external influences may impart to the flow in a laminar boundary layer a secondary motion or disturbance, which either develops further or dies out, depending on the stability of the flow. If disturbances fade away the flow is said to be stable, and it continues to be laminar. But if they grow, the flow is said to be unstable, and it can change from laminar to turbulent. The disturbances that may cause such a transition from laminar to turbulent flow are of several different kinds. Slight unevenness or obstacles on the bottom may give rise to variations of velocity and pressure which are associated with oscillations of an unstable frequency and wavelength.

This "stability theory" is primarily due to PRANDTL and his collaborators TOLLMIEN and SCHLICHTING. TOLLMIEN (1929 and 1935) in particular contributed a mathematical formulation that has made the theory of considerable practical use (Methode der kleinen Schwingungen).

TOLLMIEN (1929) calculated the frequency, wavelength, and successive increase of the amplitude of an unstable wave disturbance in flow over an even plane surface, and indicated the limit where the flow is such that the disturbances could become unstable (point of instability). He found that an undulatory disturbance is damped or amplified according to the Reynolds' number that characterises the flow and the frequency of the disturbance. If we choose as Reynolds' number $R = \frac{u_0 \delta^*}{\nu}$, where u_0 is the velocity of the undisturbed flow, δ^* is a measure of the thickness of the boundary layer,[1] and ν is the kinematic viscosity, the onset of unstable flow is given by the curve in figure 2. In the figure $\alpha = \frac{2\pi}{\lambda}$, where λ is the wavelength of the disturbance. The curve is usually called the *"neutral stability curve"*, and separates the stable region from the unstable.

[1] δ^* = the displacement thickness $= \int_0^\infty \left(1 - \frac{u}{u_0}\right) dz$, which for laminar flow is approximately $\delta/3$; where δ is the distance from the contact surface to the point where the flow velocity is 99 % of the velocity in the main stream (cf. SCHLICHTING 1951, p. 105).

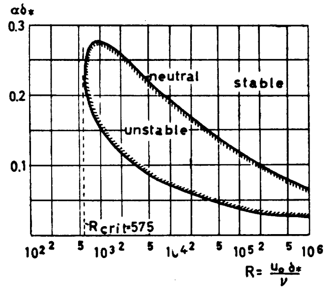

Fig. 2. The neutral stability curve for wave disturbances along an even surface as a function of the Reynolds' number (acc. to SCHLICHTING 1951).

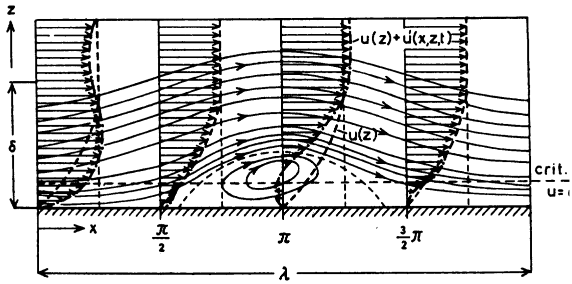

Fig. 3. Streamlines and velocity distribution for a neutral disturbance in the boundary layer along an even surface. $\lambda = 40\,\delta^* =$ the wave length of the disturbance; $c_r = 0.35\,u_0 =$ the wave velocity (acc. to SCHLICHTING 1951).

SCHLICHTING (1935 a) has calculated the velocity distribution and the shape of the flow lines for the disturbed flow for the case that falls on the neutral stability curve. For a disturbance on the curve in fig. 2 the flow shown in fig. 3 is obtained.

The stability theory provided a plausible interpretation of the physical processes that might be supposed to account for the occurrence of a critical Reynolds' number. Despite many attempts to demonstrate the existence of Tollmien's oscillatory disturbances, many years elapsed before they were finally established experimentally. In 1940 a comprehensive programme of research into the laminar-turbulent transition was commenced by the National Bureau of Standards in Washington. As part of that research SCHUBAUER and SKRAMSTAD (1947) were able to demonstrate disturbances which in their origin and development entirely conformed with the theoretical calculations. The stability theory must therefore be regarded as conclusively established.

Laminar sublayers. Surface roughness.

Even when the flow in the boundary layer is turbulent, the variable velocity components in the immediate neighbourhood of a fixed body are small. The frictional forces dominate there over the inertial forces. A very thin layer next to the surface of contact will therefore be in laminar motion. This thin layer is usually called the *laminar sublayer* (Ger. laminare Unterschicht).

A necessary condition for the formation of a laminar sublayer is that the surface over which flow takes place is so even that any projection is not higher than the thickness of the laminar sublayer. If all the irregularities lie within the laminar sublayer the friction against the underlying surface corresponds to the viscosity in the sublayer. But if the irregularities project through the laminar sublayer they offer a "form resistance" to the turbulent flow. The frictional resistance to the flow is then roughly proportional to the square of the flow velocity.

The sublayer is generally so thin that measurements to demonstrate its existence and investigate the flow within the layer are very difficult to carry out. Nor is there a sharp boundary between the laminar flow immediately adjacent to the underlying surface and the turbulent flow slightly farther away; the transition is rather gradual. Nevertheless, since the processes that determine the frictional resistance to the flow take place in this thin sublayer, it is of great significance. It has been found that the thickness of the layer is given approximately by the formula (cf. SCHLICHTING 1951, pp. 369, 412):

$$\delta_{\text{sub}} = 5 \frac{\nu}{u_*} \tag{6}$$

where $u_* = \sqrt{\frac{\tau_0}{\varrho}}$ = the *shearing stress velocity* or *shear velocity* (τ_0 = boundary shear).

As a result of experiments with flow over sand surfaces, NIKURADSE (1933, cf. also SCHLICHTING 1936 and 1951, p. 381) concluded that three different types of flow could be distinguished, corresponding to different values of the ratio of the grain size of the sand—i.e. the roughness of the surface—to the thickness of the laminar sublayer:[1]

1. *Aerodynamically smooth flow:*

$$0 \leq k/\delta_{\text{sub}} \leq 1 \tag{7a}$$

where k = the height of the projections from the surface = the diameter of the sand grains, and δ_{sub} = the thickness of the laminar sublayer $\approx 5 \frac{\nu}{u_*} = 5\,\nu\sqrt{\frac{\varrho}{\tau_0}}$.

All the irregularities (sand grains) lie within the laminar sublayer. The resistance to flow is not affected by the sand grains. The surface therefore exhibits the same resistance to flow as does a completely smooth surface.

2. *Transitional flow:*

$$1 \leq k_s/\delta_{\text{sub}} \leq 14 \tag{7b}$$

The most exposed sand grains project through the laminar sublayer. The resistance to flow increases on account of their form resistance.

3. *Fully rough flow:*

$$14 \leq k_s/\delta_{\text{sub}} \tag{7c}$$

All the irregularities project through the laminar sublayer, so that there is no longer a coherent sublayer. The resistance to flow is predominantly due to the form resistance of the sand grains, and is therefore proportional to the square of the flow velocity.

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[1] The boundaries for the different types of flow are stated somewhat differently by different authors, depending on their interpretation of experimental data. The value of δ_{sub} is often given as $11.6\ \nu/u_*$ instead of $5\ \nu/u_*$, as in (6). The coefficient 11.6 is obtained from the intersection of the curve for the logarithmic velocity distribution and the linear velocity distribution in the immediate neighbourhood of the surface. However, experimental data obtained by REICHARDT (1940) indicate that there is a modification of the flow near this point, so that the linear distribution is strictly valid only within the layer $z < 5\ \nu/u_*$.

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Boundary layer thickness and resistance

For many of the relatively simple stream-lined bodies that occur in aerodynamics and hydrodynamics it has been possible to calculate the growth and thickness of the boundary layer, the limit at which a laminar boundary layer is transformed to a turbulent one, the surface resistance, etc. In the simplest case, when the flow is over a smooth plane, the thickness of the boundary layer in laminar flow may be expressed by the formula (SCHLICHTING 1951, p. 104):

$$\delta_{\mathrm{lam}} = 5 \left(\frac{\nu x}{u_0}\right)^{0.5} \tag{8}$$

or

$$\delta^*_{\mathrm{lam}} = 1.73 \left(\frac{\nu x}{u_0}\right)^{0.5} \tag{9}$$

where x is the distance from the leading edge of the plain surface, and the other symbols denote the quantities previously stated. The transition to a turbulent boundary layer occurs for the value of x that makes $R = \frac{u_0 \delta^*}{\nu}$ so large that the flow passes into the region of instability of fig. 2.

For a turbulent boundary layer, assuming the velocity distribution to follow a seventh root power law, the thickness may be expressed by the formula (SCHLICHTING 1951, p. 394):

$$\delta_{\mathrm{turb}} = 0.37\, x^{0.8} \left(\frac{\nu}{u_0}\right)^{0.2} \tag{10}$$

or

$$\delta^*_{\mathrm{turb}} = 0.046\, x^{0.8} \left(\frac{\nu}{u_0}\right)^{0.2} \tag{11}$$

since $\delta^* = \delta/8$ for turbulent flow.

The surface resistance, which is entirely due to tangential forces in the boundary layer (boundary shear), has been found to obey the following relations, which have been verified experimentally (cf. SCHLICHTING 1951, pp. 102, 393, and ROUSE 1950, p. 106).

$$\tau_{o_{\mathrm{lam}}} = 0.33\, \varrho u_o^{1.5} \left(\frac{\nu}{x}\right)^{0.5} \tag{12}$$

$$\tau_{o_{\mathrm{turb}}} = 0.030\, \varrho u_o^{1.8} \left(\frac{\nu}{x}\right)^{0.2} \qquad (13)^1$$

[1] As previously mentioned, equations (10), (11), and (13) are derived on the assumption that the velocity distribution follows a $^1/_7$ power law. For large values of Reynolds' number ($R_x = \frac{u_0 x}{\nu} > 10^7$) it is necessary to base the calculations on the more universal logarithmic velocity distribution. SCHULTZ-GRUNOW (1940), who made detailed measurements on the flow over a smooth plate, obtained the empirical interpolation formula

$$\tau_{o_{\mathrm{turb}}} = 0.185\, \varrho u_o^2 \left(\log \frac{u_o x}{\nu}\right)^{-2.584}$$

for which the interval of validity includes even very high values of R_x.

(12) and (13) are valid if the surface is hydrodynamically smooth. For rough (sandy) surfaces and fully rough flow (cf. p. 141) PRANDTL and SCHLICHTING (1934) arrived at the following empirical formula, using experimental data obtained by NIKURADSE:

$$\tau_{o_{\text{rough}}} = \frac{\varrho u_o^2}{2}\left(2.87 + 1.58 \log \frac{x}{k}\right)^{-2.5} \tag{14}$$

where k is the diameter of the sand grains on a sandy surface. The formula applies for $10^2 < x/k < 10^6$.

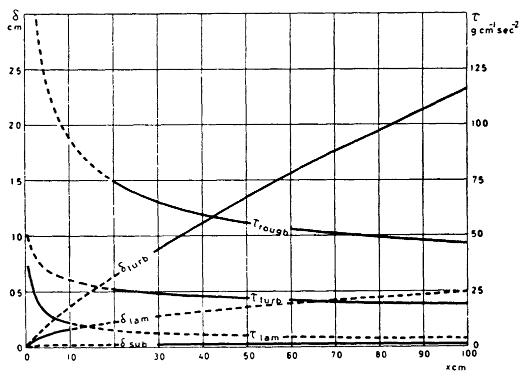

Fig. 4. Characteristics of laminar and turbulent boundary layers along a plane boundary. Thickness δ and surface resistance τ_0 as functions of the distance from the leading edge of the plate. $\nu = 0.01$ $\text{cm}^2\,\text{sec}^{-1}$, $u_o = 100$ cm sec^{-1}, $\varrho = 1$ g cm^{-3}, $k = 0.5$ cm.

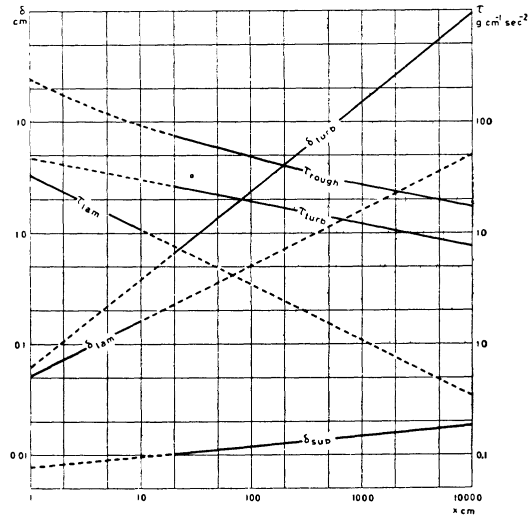

Fig. 5. The same features as in fig. 4. Logarithmic scale.

In figs. 4 and 5 the thickness and surface resistance of the various boundary layers are given as functions of the distance from the leading edge of the plate. The transition from laminar to turbulent flow may be expected to occur about 10 cm from the leading edge, when ν, u_0 and ϱ have the values stated. This transition is not sudden: it takes place in a transition zone. The transition zone has not been marked in the figures, which have been drawn on the assumption that the flow is either laminar or turbulent from the start of the boundary layer. Dashed lines indicate that the relevant state of flow there is improbable. The procedure adopted implies a certain generalisation of the actual phenomena, but this is relatively unimportant except within the transition zone.

The diagram shows that the turbulent boundary layer develops much more rapidly than the laminar, but that the surface resistance falls off more rapidly in a laminar boundary layer than in a turbulent layer. Rough flow entails greater resistance (local shear) than smooth flow.

Boundary layer separation. Form resistance

In the case of flow close to a smooth plate, described in the preceding section, the flow velocity u_0 outside the boundary layer was constant, and therefore the pressure too. But if the external flow is subjected to a pressure gradient, or if the underlying surface is locally concave or convex (which is usually the case to a certain extent in natural water-courses, since the bottom is never completely flat) the character of the boundary layer is changed.

The process may be described qualitatively as follows:" If the external flow is accelerated by a fall of pressure in the direction of the motion, the fluid particles which are travelling more slowly in the boundary layer also receive an impulse in the direction of the motion. Hence all the particles will continue on their way past the surface of the body. If, on the other hand, there is a fall of pressure in the direction opposite to that of the motion, retarding the external flow, the slower moving particles in the boundary layer are still more strongly retarded, and finally, when all their kinetic energy has been consumed, are made to turn back." (PRANDTL 1952, p. 136).

The backward flow attains a greater and greater thickness, and the external flow therefore loses its direct contact with the underlying surface. This process is called *boundary-layer separation*, and the external flow is said to be *separated* from the boundary. The situation is shown schematically in fig. 6.

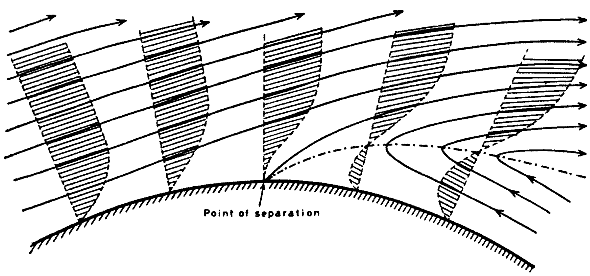

Fig. 6. Boundary-layer separation.

Boundary-layer separation leads to the occurrence of a thin layer within the flowing water, which separates regions with flows of different velocity and direction. This *layer of discontinuity* is not stable: it is rolled up to the accompaniment of large or small *eddies*.

The tendency to boundary-layer separation is greater when the boundary layer is laminar than when it is turbulent, because in turbulent flow there is a much more intense interchange of fluid between adjoining layers with different velocities, but in principle the process is the same.

An important consequence of the separation is the occurrence of the so-called *form* *resistance.* Since the point where the boundary layer leaves the underlying surface (the point of separation) is generally situated in the neighbourhood of the place where the flow velocity is greatest and the pressure least, the normal stress is low in the region of separation. For the pressure there is equal to the pressure at the point of separation, and may be regarded as almost constant over the whole region. The resultant of the various normal forces acting on the body will thus act in the direction of flow, which gives rise to a *drag,* and energy dissipation. Hence the total effect of flowing water on an immersed surface is derived from tangential stresses due to viscous shear on the one hand, and on the other from form resistance due to normal pressures.

In a natural water-course the bottom is never completely flat, even if the irregularities due to individual sand grains are neglected. Unevenness on a larger scale causes variations of velocity and pressure near the bottom, and thus variations in both viscous shear and normal pressure. As will be shown in a later section (p. 210), this is of extreme importance for the erosion and transport processes, and for details in the structure of the bottom surface (ripples, bars, etc.). It seems justifiable to regard *the flowing water in its entirety as a fully developed turbulent layer, of a thickness practically the same as the depth of the water, and the morphological processes as determined by the interaction of the bottom surface and the water's state of flow—the variations in the state of turbulence, the viscous shear, and the form resistance being dependent on the shape of the bottom and the grain size of the material there.*

An instance of how the flow depends on the bottom topography is provided by the investigations of MOTZFELD (1937), carried out on the turbulent flow near corrugated walls. He passed a current of air over surfaces with four different types of wave profile in

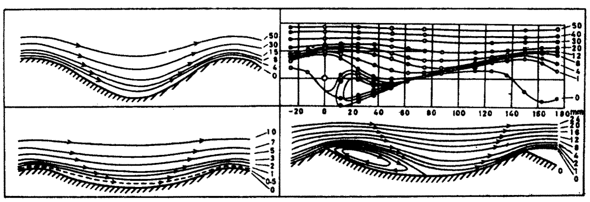

Fig. 7. Streamlines along the second and third wave profiles (upper and lower left), and streamlines and total pressure on the streamlines along the fourth wave profile (upper and lower right. According to MOTZFELD 1937).

a wind tunnel. The velocity distribution, pressure variation, form of the stream lines, and frictional resistance were determined for various wind speeds. Some of the results are presented in fig. 7. It will be seen from the diagrams how the separation at the sharp-topped wave profile gives rise to a markedly asymmetric pressure distribution, and in consequence to a large form resistance. For the trochoidal wave profile the frictional resistance was found to be of the same order of magnitude as the form resistance (ratio = 5 : 4), whereas for the sharp-topped profile the frictional resistance was far smaller (ratio = 1 : 7).

In connection with this consideration of the separation process we may also take up the question of flow in convergent and divergent channels. Experimental work was been carried out by, among others, HOCHSCHILD (1910) and NIKURADSE (1929). Nikuradse's results indicate that the thickness of the boundary layer becomes increasingly smaller as the channel becomes more convergent. On the other hand, if the channel is divergent the thickness of the boundary layer increases, the velocity of flow decreases near the wall, and at or above a certain limiting value of the angle of divergence the flow tends to leave the wall altogether. This occurs when the *half-angle* of divergence is about 5°. For angles greater than this the flow is highly unstable, there is complete separation and back flow, and the stream of water swings to and fro on account of incidental disturbances. The flow leaves the walls entirely, and a *free jet* has formed.

This phenomenon of separation plays an important part in the processes of transport and sedimentation in water-courses where the cross section is extremely variable, and particularly in the initiation and development of deltas. In a later section (p. 163) some problems associated with the formation of a jet stream are treated in somewhat more detail.

The velocity distribution near a solid boundary

The velocity distribution near a solid boundary, and therefore the velocity distribution in an open channel as well, is deducible from VON KÁRMÁN's *similarity theorem*[1]. It may be shown (the proof will not be considered here) that VON KÁRMÁN's postulate leads to a velocity distribution of the form

$$\frac{u}{u_*} = \frac{1}{\varkappa} \ln \frac{z u_*}{\nu} + C_1 \tag{15}$$

for a smooth boundary, and

$$\frac{u}{u_*} = \frac{1}{\varkappa} \ln \frac{z}{k} + C_2 \tag{16}$$

for a rough boundary, where u is the average velocity at a distance z from the boundary surface, u_* the shearing stress velocity, ν the kinematic viscosity, k the height of the irregularities projecting from the surface, and $\varkappa$, C_1, and C_2 constants ($\varkappa$ is VON KÁRMÁN's constant).

The constants in equations (15) and (16) have been determined from experiments carried out by NIKURADSE (1932, 1933) and others. The value obtained for $\varkappa$ was 0.40. Changing to ordinary logarithms and inserting the values of the constants, equations (15) and (16) become

[1] VON KÁRMÁN (1930) assumed that, if Reynolds' number was sufficiently large for the effect of the water's viscosity to be neglected, turbulent mixing took place in a similar manner everywhere. The turbulent mixing could therefore be characterised solely by a unit of length and a unit of time. von Kármán then arrived at the result that the length parameter l (the PRANDTL "mixing length") is equal to $\varkappa \frac{du}{dz} \Big/ \frac{d^2u}{dz^2}$, where $\varkappa$ is a constant, called von Kármán's constant (cf. PRANDTL 1952, p. 129).

$$\frac{u}{u_*} = 5{\cdot}75 \log_{10} \frac{z u_*}{\nu} + 5{\cdot}5 \text{ (smooth boundary)} \tag{17}$$

and

$$\frac{u}{u_*} = 5{\cdot}75 \log_{10} \frac{z}{k} + 8{\cdot}5 \text{ (rough boundary)} \tag{18}$$

k is the diameter of sand grains on a sandy surface. The boundaries between aerodynamically smooth flow, transitional flow, and fully rough flow are indicated by (7 a), (7 b), and (7 c) (p. 141).

For transitional flow the velocity distribution may be represented by the equation

$$\frac{u}{u_*} = 5{\cdot}75 \log_{10} \frac{z}{\alpha k} + 8{\cdot}5 \text{ (transitional flow)} \tag{19}$$

where α is a correction factor, with a value that depends on the ratio of k to the thickness of the laminar sublayer ($\alpha = \psi\, [k/\delta_{\text{sub}}]$).

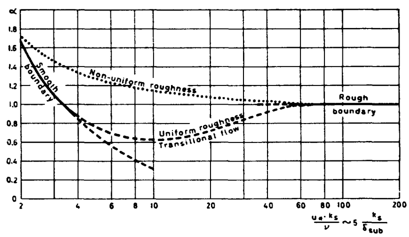

Fig. 8. The function $\alpha = \psi\, [k/\delta_{\text{sub}}]$.

The function $\alpha = \psi\, [k/\delta_{\text{sub}}]$ is illustrated in fig. 8, which has been constructed with the help of NIKURADSE's experimental results (cf. SCHLICHTING 1951, p. 383, and EINSTEIN 1950, fig. 4).

Equation (19) may also be written in the alternative forms

$$\frac{u}{u_*} = 5{\cdot}75 \log_{10} \frac{z}{\alpha \cdot k} + 8{\cdot}5 = 5{\cdot}75 \log_{10} \left(\frac{30.2\, z}{\alpha \cdot k} \right) \tag{20}$$

So, if the flow velocity u_1 at a height z_1 above the bottom is known, the flow velocity at any level z may be calculated by means of the formula

$$\frac{u}{u_1} = \frac{\log_{10} \left(\frac{30.2\, z}{\alpha \cdot k} \right)}{\log_{10} \left(\frac{30.2\, z_1}{\alpha \cdot k} \right)} \tag{21}$$

According to fig. 8, α for rough flow is 1. The value of u is then 0 for $z = z_0 \approx k/30$, that is to say, the theoretical zero velocity for a logarithmic velocity distribution occurs somewhat above the theoretical boundary surface. No measurements have been made which directly refer to individual sand grains, but according to measurements by EINSTEIN and EL-SAMNI (1949) equation (21) is valid at least as near the theoretical boundary surface as 0.1 k, and the assumption that the turbulent form of the velocity distribution is applicable as far as the zero velocity level appears to be at least as justified as any other.

The value z_0 of z is often called the roughness parameter of the surface (cf. DEACON 1953, p. 13).

The velocity distribution is sometimes given as a power function (cf. p. 142 and PRANDTL 1952, p. 127). The seventh root power formula provides a good approximation to the actual velocity distribution for relatively small Reynolds' numbers. For larger R eighth, ninth, and tenth root power formulas may be used with advantage. But all power formulas are no more than approximations for the more universal logarithmic velocity distribution of equations (17)—(21).

The data on which the preceding formulas have been based were obtained from experiments using pipes, or plane surfaces coated with sand of uniform grain size. The conditions in natural water-courses are of course more complicated, but field investigations show that the same equations may be applied even there. The logarithmic velocity distribution must therefore be regarded as the best approximation to the real vertical distribution of the average flow velocity, with obvious exceptions where there are irregularities in the topography of the bottom, secondary flow, or other sources of disturbance.

The preceding formulas have assumed that there is a uniform distribution of grain size, and the roughness of the surface has been represented by the grain diameter k. In natural water-courses the grain size is often far from uniform, and the question arises as to what is a representative value of k for a particular bottom sediment. According to flume experiments carried out in the laboratory by EINSTEIN (1950, p. 8), "the representative grain diameter of a sediment mixture is given by that sieve size of which 65 per cent of the mixture (by weight) is finer". This empirical result makes it possible to use the formulas involving k even for non-uniform material, but assumes that the sorting of material is not poor, as well as that the bottom is fairly level without bars and ripples.

If there are obstacles of some kind on the bottom—bars, ripples, scattered stones, or vegetation, the grain diameter does not constitute a representative value for the roughness, and the parameter z_0 can no longer be set equal to $k/30$. SCHLICHTING (1936), and later other investigators such as ROUSE (1943) and EINSTEIN and BANKS (1950), have carried out experiments on flow over surfaces with other kinds of roughness than that due to uniformly distributed sand grains. SCHLICHTING in his experiments determined the value of the surface's *equivalent sand roughness*. Equivalent sand roughness denotes the roughness of a hypothetical surface made of tightly packed sand, with the same effect on the vertical flow profile as the actual surface.

If k denotes the height of the roughness element in a surface, the equivalent sand roughness may be expressed by the equation (cf. SCHLICHTING 1951, p. 385, and KEULEGAN 1938, p. 722)

$$5.75 \log_{10} \frac{k_s}{k} = 8.5 - C \qquad (22)$$

where C is a constant, corresponding to the constant C_2 in equation (16), and k_s is the equivalent sand roughness.

As has already been pointed out in the section on boundary-layer separation and form resistance (p. 145), the resistance to flow exerted by a corrugated surface is a combination of the viscous frictional resistance and the form resistance. This is important when considering the transport of sediment over a sandy surface where ripples or bars have formed. The eddies or wakes that develop on the lee of each ripple or bar cause pressure differences along the direction of flow and a definite shape resistance arises. Turbulence is generated, but the corresponding energy transformation does *not* contribute to the transport of sedimentary particles. The transport process is maintained solely by the true frictional resistance, which acts on each individual sand grain. This distinction was pointed out by EINSTEIN (1950, p. 9) in his important work on bed-load transportation. The problem is considered in more detail in the section on the critical erosion velocity (p. 182).

In the technical literature the flow velocity in a channel or water-course is often given in terms of the slope (more exactly, the slope of the energy grade line), and the depth, or the hydraulic radius. Formulas concerned may be obtained from the standard textbooks on hydraulics.

Velocity fluctuations

Where the surface resistance and the velocity distribution have been mentioned in previous sections the velocity referred to has always been the average velocity over a certain time interval. The formulas adduced therefore apply to time averages, and not to instantaneous values. It is in the nature of turbulence that the general flow movement has superposed upon it a complex secondary motion with apparently random variations.

The morphological processes—erosion, transport, and sedimentation—cannot be satisfactorily explained without considering these velocity fluctuations. Information about the magnitude and character of the velocity fluctuations in natural water-courses indicates the state of turbulence in the flowing water, and is thus a necessary complement to the ordinary average values. However, at the present position of research it is unfortunately very difficult to carry out field measurements on the rapid fluctuations, and moreover, difficulties encountered in the interpretation of the results are also considerable. The description of the flow process is at present usually presented in a statistical guise, which is well suited to its purpose but nevertheless veils some of the details of the actual physical process.

If the variable velocity vector at a particular point is represented by the three components u, v, and w, parallel to the three coordinate directions x, y, and z, the following relationships are valid for the instantaneous values of the flow velocity:

$$u = \bar{u} + u'; \quad v = \bar{v} + v'; \quad w = \bar{w} + w'; \tag{23}$$

where $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the average values, and u', v', and w' are the superposed, fluctuating components of the velocity. Similarly, for the variable pressure in the fluid,

$$p = \bar{p} + p' \tag{24}$$

Since $\bar{u}$, $\bar{v}$, $\bar{w}$, and $\bar{p}$ are defined as time averages, $\left(\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)dt\right)$, the arithmetic means of u', v', w', and p' are zero, i.e. $\bar{u}' = \bar{v}' = \bar{w}' = \bar{p}' = 0$.

The expressions $\sqrt{\overline{u'^2}}$, $\sqrt{\overline{v'^2}}$, and $\sqrt{\overline{w'^2}}$ represent the statistical standard deviation σ for the fluctuating velocity components. The ratio $\frac{\sqrt{\overline{u'^2}}}{\bar{u}}$ is a measure of the relative magnitude of the velocity fluctuations in the x direction. Assuming the axes of coordinates to be so chosen that $\bar{v} = \bar{w} = 0$, i.e. the average flow is parallel to the x axis, the following expression may be used as a measure of the degree of turbulence

$$\frac{\sqrt{\frac{1}{3}\left(\overline{u'^2}+\overline{v'^2}+\overline{w'^2}\right)}}{\bar{u}} = \text{"intensity of turbulence"} \qquad (25)$$

A large number of measurements of the velocity fluctuations in air have been made, both in wind tunnels and under natural conditions, beginning with W. SCHMIDT's fundamental work on the structure of the wind in the air layers nearest the ground (SCHMIDT 1929 a and b). Recently the hot-wire anemometer, and, latest, thermistor apparatus, have been applied to the problem. But the measurements in flowing water are much fewer, partly due to the fact that the hot-wire anemometer is not appropriate for measurements in water.

KALINSKE (1940) was able to obtain an approximate value of the transverse component of the velocity fluctuations, v', by injecting a fine jet of dark red liquid into the flowing water in a flume and filming the spreading of the coloured liquid at a short distance from the point of injection. By injecting droplets of a mixture of carbon tetrachloride and petrol, which were illuminated and filmed, it was also found possible to determine the components u' and v', from the length and direction of the individual streaks on the film.

KALINSKE (1943) has also investigated velocity fluctuations in the Mississippi. He used an ordinary "Price current meter so arranged that the time required for making a single revolution of the cup-wheel was recorded". As he mentions, a current meter cannot of course register the really rapid fluctuations, nor can it distinguish between different components of the velocity, but the results obtained with it are nevertheless a valuable contribution to our knowledge of turbulence in rivers.

KALINSKE (1943, p. 269) carried out measurements at three levels in the river. He then investigated the frequency of the random fluctuations $u' = u - \bar{u}$ in different velocity intervals, and thereby obtained typical frequency distributions of which fig. 9 is an example. The standard deviation was computed, and the relative intensity of the turbulence could then be calculated for different depths. In this connection it should be observed that the intensity of turbulence is in this case not exactly the same as in formula (25), since the different components of the fluctuations could not be distinguished in the method used. Fig. 10 shows values of the intensity of turbulence in nine different sections in the river.

The results indicated in figs. 9 and 10 are in general agreement with the evidence of laboratory experiments. They give rise to several interesting conclusions regarding the nature of the fluctuations:

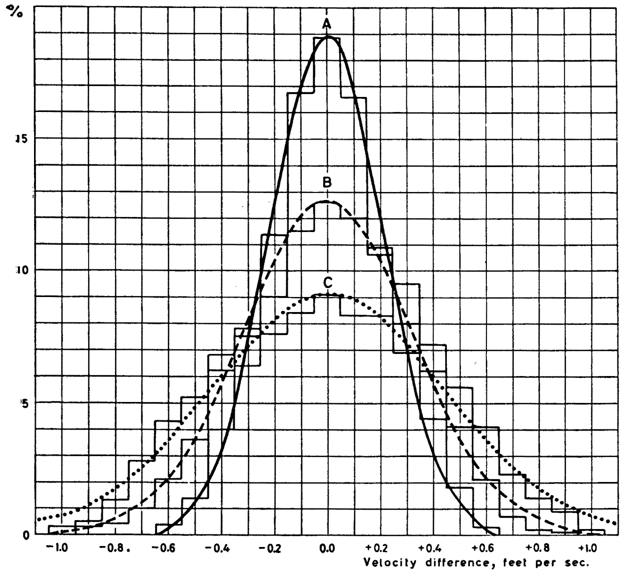

Fig. 9. Frequency of velocity fluctuations at different levels above bottom. Block diagrams and normal error curves. A: 15.2 feet above bottom, $\overline{u} = 3.27$ feet per sec, $\sigma = 0.212$ feet per sec. B: 7.6 feet above bottom, $\overline{u} = 2.85$ feet per sec, $\sigma = 0.316$ feet per sec. C: 1.5 feet above bottom, $\overline{u} = 2.10$ feet per sec, $\sigma = 0.437$ feet per sec (redrawn from KALINSKE 1943).

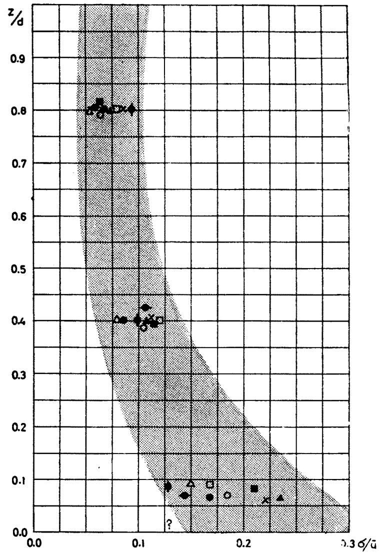

Fig. 10. Variation of relative intensity of turbulence with relative depth in nine sections in Mississippi River. σ = standard deviation of velocity, $\overline{u}$ = mean velocity, d = total depth, and z = distance above bottom (acc. to KALINSKE 1943).

1. The magnitude of the fluctuations increases from the surface downwards. The flow is thus more irregular near the bottom.[1]

2. The velocity fluctuations fit quite well a normal error curve, so that "fully developed turbulence appears to be a haphazard velocity fluctuation".

3. According to the frequency curves the maximum of u' (equal to $u - \overline{u}$) was equal to about 3σ.

4. It may be seen from fig. 10 that the general variation of the relative intensity of turbulence at different depths is similar for the different sections, though the spread of the values is quite large.

KALINSKE (1943, p. 274) concludes from theoretical reasoning that the relative intensity of turbulence "depends on the relative roughness of the bottom and the Reynolds' number of the flow". He further observes that "fluctuations equal to twice the mean velocity at a point in the turbulent zone near a boundary can readily be expected".

Since the intensity of turbulence is of vital importance as regards the transport of fluviatile sediments, systematic investigations of the turbulent fluctuations in various

[1] REICHARDT (1938) carried out measurements of the velocity fluctuations in a wind tunnel, and found that both $\sqrt{\overline{u'^2}}$ and $\sqrt{\overline{v'^2}}$ attain maxima close to the wall. For fluctuations in the direction of flow the maximum was very marked, while for the transverse fluctuations $\sqrt{\overline{v'^2}}$ was more evenly distributed.

types of water-course, with different bottom configurations and various orders of size, would be particularly valuable.

The work of KALINSKE and others has shown that the velocity fluctuations in natural water-courses are random, and that they follow a normal error distribution. These conclusions are based on measurements at a relatively large distance from the bottom. It might be suspected that conditions are somewhat different in the *immediate* neighbourhood of a rough surface.

Laboratory investigations by EINSTEIN and EL SAMNI (1949) indicate that this is actually the case. From measurements of the static and dynamic pressures at an artificially rough surface it was found that the pressures at the wall were "statistically distributed according to the normal error law. One is tempted to conclude from this fact, that the pressures are the basic variables in the statistical description of turbulence in the vicinity of a wall, rather than the velocity" (p. 523).

Since the pressure fluctuations follow a normal error curve, according to EINSTEIN and EL SAMNI, the velocity distribution in the immediate vicinity of a wall must be skew, since a fully symmetric distribution would be incompatible with the observed pressure distribution (EINSTEIN 1950, p. 13).

Turbulent exchange

The irregular velocity fluctuations of turbulent flow are due to the interchange of particles or "balls" of liquid between layers of liquid with different velocities. A rough qualitative description of the process may be given as follows.

Owing to the formation of vortices and eddies near solid boundaries and surfaces of discontinuity, the flow is continually subject to new disturbances. The vortices move with the current, undergo modification and deformation, and finally die out. This motion is accompanied by a transverse motion as well, and the latter promotes internal mixing in the flowing fluid. The vortices or turbulence "bodies" that move from layers of lower to layers of higher velocity cause a retardation of the flow in their new environment, while, conversely, vortices moving in the opposite direction cause a certain acceleration. This *turbulent exchange* therefore gives rise to a shearing force between neighbouring layers of fluid, and thereby determines the vertical distribution of the average velocity, which has been considered in an earlier section.

Turbulent exchange must be regarded as one of the most important aspects of turbulent flow. For fluvial morphology it is of particular importance in the discussion of the transport of the suspended material.

Because of the complicated nature of turbulent flow it has not yet been possible to attain a full understanding of it, neither experimentally nor theoretically, and even for the average state of flow there is as yet no theory quite free from objections. In the theoretical treatment it has been found necessary to start from semi-empirical premises, generally a combination of the observed average velocity distribution and the shearing stress that arises from the turbulent exchange.

As already mentioned (p. 134), BOUSSINESQ (1877) was the first to relate the apparent viscosity due to turbulent exchange and the velocity gradient. Since BOUSSINESQ a great deal has been published on turbulent exchange, particularly in the field of meteorology. Among the more important earlier publications are those of ÅKERBLOM (1908), TAYLOR (1915), RICHARDSON (1920), SCHMIDT (1925), SUTTON (1932), and KÖHLER (1933). In the literature on fluviatile processes the theory of turbulent exchange was first employed by HJULSTRÖM (1932 and 1935), LEIGHLY (1932 and 1934), O'BRIEN (1933), and CHRISTIANSEN (1935). A great deal has been published since the middle thirties. With unimportant exceptions these publications all use much the same conception of the fundamental processes involved.

PRANDTL (1925, p. 137) gives the following simplified account of the mechanism of turbulent flow. In a turbulently flowing liquid there occur small bodies of liquid that have a fairly independent motion over a certain distance, until they lose their identity by mixing with the surrounding turbulent medium. This representative distance l was called by PRANDTL the *mixture length* or *mixing length* (Ger. Mischungsweg).[1] By applying the momentum theorem for a flow with fluctuating velocity it can be shown that the apparent shearing stress between contiguous layers is given by

$$\tau = \varrho\, l^2 \left|\frac{d\bar{u}}{dz}\right| \frac{d\bar{u}}{dz} \tag{26}$$

where $\left|\frac{d\bar{u}}{dz}\right|$ signifies the absolute value of the change of the average velocity with height above the bottom.

This formula, which is usually referred to as the Prandtl mixing-length formula, has been successfully applied both to flow past boundary surfaces and to free turbulence, i.e. turbulence far from a solid boundary—for instance, turbulence in the free atmosphere or in the sea. However, it has the disadvantage that it implies zero values for the apparent shearing stress and the turbulent exchange at places where $\frac{d\bar{u}}{dz} = 0$. This would mean that turbulent exchange in the vertical direction must be zero near the surface in flowing water, which evidently does not conform with the real state of affairs (cf. p. 215). Moreover, the utility of the formula is limited as long as it is not possible to state l for a particular case of turbulent flow as a function of the position coordinates.

VON KÁRMÁN (1930) attempted to overcome the latter difficulty. He assumed (cf. note p. 146) that the mechanism of turbulent flow is such that the turbulent motion or the turbulent mixing at large values of Reynolds' number takes place in a similar manner for two different but geometrically similar flow configurations. The turbulent exchange can

[1] "Die gesuchte Länge l ist nun dadurch charakterisiert, dass sie die Entfernung von der betrachteten Schicht angibt, in der die durchschnittlichen u-Geschwindigkeiten, die die Flüssigkeitsballen bei ihrem Durchtritt haben, als zeitlicher Mittelwert der Strömungsgeschwindigkeit angetroffen werden . . . Dass l der Grösseranordnung nach mit dem Durchmesser des Flüssigkeitsballens übereinstimmt, sei nebenher erwähnt (genauer ist es der "Bremsweg" des Flüssigkeitsballen in der übrigen Flüssigkeit, der aber dem Durchmesser proportional ist)."

then be characterised solely by a unit of length and a unit of time (VON KÁRMÁN's *similarity theorem*). The mixing length l is chosen as the unit of length, and for the unit of time (really the unit of velocity) the turbulent shearing stress velocity $u_* = \sqrt{\frac{\tau}{\varrho}}$.

Mathematical development of the similarity hypothesis leads to the following expression for the mixing length:

$$l = \varkappa \left| \frac{\frac{d\bar{u}}{dz}}{\frac{d^2\bar{u}}{dz^2}} \right| \tag{27}$$

where $\varkappa$, von Kármán's constant, is an empirically determined dimensionless constant, the same for all turbulent flow.[1]

The similarity hypothesis yields the same formula for the apparent shearing stress as equation (26), and if l from (27) is inserted in (26) one obtains

$$\tau = \varrho \varkappa^2 \frac{\left(\frac{d\bar{u}}{dz}\right)^4}{\left(\frac{d^2\bar{u}}{dz^2}\right)^2} \tag{28}$$

For two-dimensional flow it is easily shown (cf. SCHLICHTING 1951, p. 356), that the shearing stress at a level z above the bottom may be given by

$$\tau = \tau_o \left(1 - \frac{z}{z_m}\right) \tag{29}$$

where τ_0 is the shearing stress at the boundary, and z_m the height above the bottom at which the flow velocity is a maximum (in general z_m is approximately equal to the depth of water).[2]

Combination of (28) and (29), together with double integration leads to the logarithmic velocity distribution.[3]

Comparison of BOUSSINESQ's formula (equation (3), p. 134) and (25) gives the following value for A (the "Austauschkoeffizient")

[1] The value of $\varkappa$ is usually given as 0.40, sometimes slightly lower.

[2] Frequently τ is treated as a constant ($= \tau_0$) in a particular layer. This is permissible, for instance, in considering the turbulent diffusion in atmospheric layers near the ground (DEACON 1953, p. 12, SUNDBORG 1955, p. 98), or in a thin layer near the bottom in a water-course where the flow is treated as two-dimensional, but it leads to an erroneous conception of the distribution of the sediment in a vertical section if the simplification is extended over the total depth of water.

[3] Actually, this integration does not yield the logarithmic velocity distribution in the same form as equations (15) and (16), because these equations were deduced on the assumptions that l is proportional to z and τ = const. In spite of the different assumptions involved, however, equations (15) and (16) give a very good approximation for the *velocity distribution*, since the greatest part of the velocity change occurs in the immediate neighbourhood of the boundary. (15) and (16) have therefore been employed, on account of their simple and adequate formulation.

$$A = \varrho l^2 \left| \frac{d\bar{u}}{dz} \right| \tag{30}$$

or, for ε $(= A/\varrho)$, the *momentum transfer coefficient* or *eddy viscosity coefficient.*

$$\varepsilon = l^2 \left| \frac{d\bar{u}}{dz} \right| \tag{31}$$

The apparent shearing stress is customarily expressed in the form

$$\tau = A \frac{d\bar{u}}{dz} = \varrho\varepsilon \frac{d\bar{u}}{dz} \tag{32}$$

Formula (32) expresses how the turbulent mixing gives rise to an apparent shearing stress by the transfer of momentum between contiguous layers of flow. But the process of exchange involves a transfer between adjacent layers of other things besides momentum. Temperature contrasts and contrasts in the concentrations of dissolved salts or suspended particles are also evened out by the turbulent mixing.

By analogy with the formulas for the transfer of momentum the amount of substance transferred per unit time across unit area perpendicular to the concentration gradient for the substance at the point in question may be written in the following manner for two-dimensional flow:

$$S = -A \frac{ds}{dz} = -\varrho\varepsilon \frac{ds}{dz} = -\varrho l^2 \left| \frac{d\bar{u}}{dz} \right| \frac{ds}{dz} \tag{33}$$

where S denotes the amount of substance transferred per unit area per unit time, and s the concentration of substance per unit volume.

However, it cannot be assumed that the Austauschkoeffizient A or the eddy viscosity coefficient ε are identical in formulas (32) and (33). It is in fact probable that the mechanism involved is not entirely the same for the transfer of momentum as for turbulent diffusion of dissolved or suspended material or turbulent conduction of heat. The microstructure of the turbulent flow is of great importance in this connection, especially the orientation of the axes of the individual eddies (cf. PRANDTL 1952, p. 120). Furthermore, in the diffusion of suspended matter, e.g. particles of sediment, the mass of the solid particles enters into the process, since relatively heavy particles cannot, owing to their inertia, follow all the rapid fluctuations in highly turbulent flow.

This problem will be considered in the section on the nature of turbulence (p. 161) and the transport of material (p. 214). However, even at this stage it may be remarked that the transfer of solid matter or heat is analogous to the transfer of momentum, though it is uncertain whether the coefficients have exactly the same numerical value.

Accordingly, it is both from a theoretical and from a practical point of view convenient to use different symbols, connected by the relation (cf. ISMAIL 1951, p. 2)

$$\varepsilon_s = \beta \cdot \varepsilon_m \tag{34}$$

where ε_s is the transfer coefficient for matter (sediment), ε_m the transfer coefficient for momentum, and β a factor of proportionality which is generally close to unity.

According to (32), ε_m may be written $\varepsilon_m = \dfrac{\tau}{\varrho \dfrac{d\bar{u}}{dz}}$. Taking (29) and (34) into consideration this leads to

$$\varepsilon_s = \frac{\beta \cdot \tau_o}{\varrho \cdot \dfrac{d\bar{u}}{dz}} \left(1 - \frac{z}{z_m}\right) \tag{35}$$

(35) gives the transfer coefficient for matter as a function of the boundary shear, the flow velocity gradient, and the position coordinates in two-dimensional flow. If $\dfrac{d\bar{u}}{dz}$ is obtained by differentiating the formula for the vertical distribution of the flow velocity, ε_s is obtained as a function solely of the position coordinates. It should then be possible to compute, for instance, the vertical distribution of suspended sediment in a water-course. Such a calculation is presented in the section on the movement of suspended material (p. 213), where the question of how well the theoretical conclusions agree with experimental results on suspended material is also discussed.

Density stratification. Density currents

In the preceding sections it has been assumed that the flowing water has uniform density. If, however, there are differences of density between adjoining layers, or if there is a more or less continuous change of density with distance from the bottom, the stability of the flow will be affected. Supposing that the density increases towards the bottom, the stability is enhanced, the turbulent velocity fluctuations—especially in the vertical direction—are damped or prevented, and the turbulent exchange between contiguous layers becomes weaker or ceases. This is a consequence of the fact that transfer of heavier liquid upwards and of lighter liquid downwards requires more energy as density differences increase. Not only the turbulent exchange is affected, but also the vertical velocity distribution, the shearing stress, and, for flow with a Reynolds' number near the critical value, the transition from laminar to turbulent flow.

Appreciable density differences occur in flowing water when the content of suspended material is high, since the concentration is greatest at the bottom and decreases upwards, particularly for coarser particles (cf. p. 214). At the junction of two water-courses, or where a water-course runs into a lake, the difference in the temperatures of the two masses of water may entail a sufficiently large density difference for the turbulent mixing to be affected. Differences in the content of dissolved salts has the same effect, of course. During recent years it has become increasingly evident that such variations of density constitute a factor of considerable significance in determining the state of flow in water bearing a load of sediment. This is perhaps especially marked where a stream of sediment-bearing water flows out into or through a lake or other broad body of water, and therefore to a high degree in the building of the distal parts of a delta or for sedimentation in the more distant parts of a lake.

Theoretical discussion of the significance of density stratification was first taken up by RICHARDSON (1920 and 1925). Later the problem was considered by PRANDTL (1929), TAYLOR (1931 a and b), GOLDSTEIN (1931), and SCHLICHTING (1935 b), and others.

Consideration of the energy balance of turbulent motion in a flow with density stratification reveals that it is possible to derive a dimensionless constant, which, together with Reynolds' number, characterises the stability of the flow. This constant represents the ratio of buoyancy forces to frictional forces, and has the following form

$$R_i = -\frac{g}{\varrho}\frac{d\varrho}{dz} \Big/ \left(\frac{d\bar{u}}{dz}\right)^2 \tag{36}$$

where ϱ is the density of the liquid. R_i was first presented in this form by PRANDTL (1929), but since the basic conceptions involved occurred already in RICHARDSON's publications, R_i has been named *Richardson's number* (cf. PRANDTL 1952, p. 382).

Where R_i is zero the liquid is homogeneous, without variations of density, and the stratification may be described as *neutral*. For $R_i > 0$ the stratification is *stable*, and for $R_i < 0$ it is *unstable*.

Different investigators have stated different values of R_i for the transition from turbulent to laminar flow. Early investigations (PRANDTL 1929 and TAYLOR 1931 a) gave the values 2 and 1 respectively, while other theoretical calculations (TAYLOR 1931 a, and GOLDSTEIN 1931) yielded a critical value of 0.25. Application of TOLLMIEN's theory on the stability of a laminar boundary layer adjoining a solid surface (see p. 139) enabled SCHLICHTING (1935 b) to calculate the critical value of R_i for flow past a plane boundary surface. He arrived at $R_i = 0.04$ as the value for the transition from laminar to turbulent flow. SCHLICHTING's theoretical result is in good agreement with the experimental results of REICHARDT (PRANDTL and REICHARDT 1934) for the initiation of turbulence in a wind tunnel where the upper wall was heated and the lower cooled.

For free flow far from boundary surfaces TAYLOR (1931 b) observed turbulence at values of R_i considerably larger than 0.04. Lack of experimental data and field observations renders it impossible at present to state a definite critical value of R_i. It seems correct to state the present position of research as PRANDTL (1952, p. 382) has done: "Until further information is available, therefore, the limit of stability for free flow may be taken as $R_i = 0.25$". For flow close to a solid boundary the critical value of R_i may fall to as low as 0.04.[1]

From the viewpoint of sedimentology it is not only the density stratification at which turbulence ceases that is of importance, but also what effect a particular sub-critical density stratification may have on the velocity distribution and the turbulent exchange. It may be expected a priori that the density stratification occurring, for instance, in a water-course with high transport of sediment, will affect the turbulence to some degree,

[1] AXELSSON (1956) states that a density contrast of 0.0003 between inflowing river water and entraining lake water is sufficient to suppress turbulence and mixing between the two bodies of water in lake Laitaure in northern Sweden.

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the change being such that the turbulent exchange is weakened and the transport capacity of the flow diminished.

Experimental investigations by VANONI (1953) indicate that the exchange coefficient ε_s does actually decrease as the concentration of sediment increases. But exact measurements are too few to permit the formulation of a definite relationship between the density stratification and the exchange coefficient.

In the lower layers of the atmosphere a more or less analogous situation arises when there is a marked ground temperature inversion. A pronounced density stratification develops on account of the temperature stratification, and the air flow becomes stable. SVERDRUP (1936) has analysed the change in the eddy viscosity with velocity and temperature profiles when there is stable temperature stratification, and he makes use of an equation that expresses the dependence of the velocity change on Richardson's number:[1]

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\varkappa z}\left(1 + \sigma R_i(z)\right)^{1/2} \tag{37}$$

where $R_i(z)$ is Richardson's number at a height z above the bottom, and σ a factor of proportionality.

Comparison of (37) with (15) or (16) shows that (37) differs from the equations obtained by differentiating (15) and (16) by the factor $(1+\sigma R_i(z))^{1/2}$. This factor may thus be regarded as corresponding to the effect of the density stratification.

DEACON (1953, p. 55) has compared equation (37) with the results of his own and others' measurements. He found that σ varies in an irregular manner and is therefore not a universal constant. A less ambiguous result is obtained by using an equation which was first suggested by HOLZMAN (1943), and which closely resembles that of ROSSBY and MONTGOMERY:

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\varkappa z}\left(1 - \sigma_H R_i(z)\right)^{-1/2} \tag{38}$$

According to DEACON (p. 56), "there is no systematic variation of σ_H with stability or surface roughness. The mean value, excluding the two points at the extremes of the stability range, is 7.1, with a standard error of 0.6".

It seems likely that a relationship of the same type as (38) may apply to the case of a sediment-bearing stream of water, although σ_H need not be expected to have the same value in such a case.

In accordance with (35), the transfer coefficient for matter, ε_s, is inversely proportional to $d\bar{u}/dz$. If $d\bar{u}/dz$ is eliminated from (35) and (38) we obtain for ε_s:

$$\varepsilon_s = \beta \cdot u_* \varkappa z \left(1 - \frac{z}{z_m}\right)\left(1 - \sigma_H R_i(z)\right)^{1/2} \tag{39}$$

If R_i is estimated from some of the laboratory experiments performed by VANONI 1953), and if σ_H is taken as 7, values of ε_s are obtained which are 40—50 % lower in certain cases compared to flow with no density stratification.

[1] This equation was previously put forward by ROSSBY and MONTGOMERY (1935).

Equation (39) cannot of course be regarded as conclusive until it has been verified experimentally. But it is nevertheless possible that it corresponds to a process that may account for some of the differences in the interpretation of laboratory and field investigations of the effect of suspended material on the state of flow which have appeared in some recent American articles (ISMAIL 1951, LAURSEN, CARSTENS, and ISMAIL 1952, VANONI 1953, and MITCHELL, LAURSEN, RAND, and VANONI 1953).

The effect of density stratification in damping turbulent exchange has come most to the fore in the discussion of so-called *density currents*. According to a definition by BELL (1942), *"a density current may be described as a gravity flow of a liquid or a gas through, under, or over a fluid of approximately equal density"*.

Although scattered observations of density currents have been reported long ago (cf. FOREL 1895), it is not until quite recently that density currents have been commonly discussed in the hydrodynamical and geological literature. The reason that little attention had been given to them earlier appears to be that they are not directly visible in general, since they are concealed beneath the undisturbed surface of a river, lake, reservoir, or ocean.

It called considerable remark in the middle 1930's when the statement appeared that turbid water from the Colorado River seemed under certain conditions to flow through the reservoir Lake Mead above the Boulder Dam practically unmixed with the surrounding water, and that in doing so it probably followed the old channel of the Colorado River. GROVER and HOWARD (1938, p. 720) wrote, "Turbid water carrying a considerable load of fine silt was discharged from Lake Mead, above Boulder Dam, in Arizona and Nevada, at three different periods during 1935 when the reservoir was 70 to 90 miles long and contained from 4,000,000 to 5,000,000 acre-ft of water. Apparently, it flowed through the reservoir essentially unmixed . . . The phenomenon is ascribed to the greater specific gravity of the incoming water relative to the generally clear water at the surface of the lake, due probably, in part, to its silt load".

Although similar observations had been published earlier, this instance from Lake Mead aroused great interest, and various other instances of density currents were thereupon put forward. The not uncommon phenomenon that the streams of two merging rivers may continue to run parallel to one another in their common channel for a considerable distance without mixing was described from several places. One contribution to the discussion following upon the article by GROVER and HOWARD mentioned a remarkable case where the water from a tributary of the Rio Grande flowed unmixed under the main stream and subsequently came to the surface on the other side of it (BLISS 1938, p. 748): . . . "On one occasion, the line of demarcation between the two flows was described as quite noticeable, being almost as distinct as if they had been separated mechanically. It was further stated that such tributary waters might appear on the right, or entrance, side of the channel, or they might pass under the main stream and appear on the opposite side".

Both these phenomena, exemplified by the density current through Lake Mead and the parallel streams in the Rio Grande, imply that an interface or surface of discontinuity forms between the two bodies of water with different densities. The change in the density across this interface is so large—and accordingly Richardson's number as well—that ordinary turbulent mixing does not occur.

The persistence of a density current will depend partly on the conditions governing mixing with the adjoining water, and partly on the state of flow of the density current.

As just mentioned, mixing between the two bodies of water does not occur as turbulent mixing—or, at least, only to a very small extent. As KNAPP (1943, p. 294) has pointed out, and partly verified by means of laboratory experiments, two types of mixing can be distinguished: a) localised mixing, and b) general mixing across the boundary or interface separating the two fluids.

Localised mixing can occur in the zone or at the point (the plunge point) where the density current leaves the surface and continues as an undercurrent in a reservoir or lake, or where the density current is hindered by an obstacle on the bottom, which leads to the liberation of sufficient kinetic energy for mixing to a greater or less degree.

The general mixing across the interface "is a wave phenomenon and is governed by the normal laws of surface waves applied to the physical conditions of the density flow" (KNAPP 1943, p. 299). The theory for the initiation of wave motion in the boundary layer between two media of different densities was first formulated by THOMSON. An account of this theory is given by LAMB (1932, p. 461). Mixing of the two media commences when the waves are unstable, in which case they increase in amplitude and finally break. When a wave breaks portions of the two media mix together.

The state of turbulence in a density current determines the distribution of grain size in the suspended sediment. If a density current has a higher content of suspended matter than the surrounding water—a *turbidity current*—a decrease in the degree of turbulence may lead to sedimentation, whereupon the flow loses its character of density current. The greater the proportion of large-grained material in suspension the more sensitive the density current is to changes in the turbulence.

There remain many unsolved problems regarding the complex interrelations of density difference, state of flow, initiation of waves, sedimentation, etc. KUENEN has in a series of articles demonstrated the importance of density currents for the origin of graded sediments (1950 a), for the formation of submarine canyons (1950 b), and for the deposition of glacial varves (1951 a). In yet another article (1951 b), he gives an account of experiments on the properties of density currents and their rôle of geological agent. BATES (1953), in connection with an investigation of the conditions for delta formation, has pointed out the influence of density currents on the character of jet streams (cf. p. 164). In an investigation at present in progress in the Sarek region in Sweden, the occurrence of density currents has been established and their significance verified (AXELSSON 1955). A symposium of the Society of Economic Paleontologists and Mineralogists on turbidity currents and the transportation of coarse sediments to deep water was held at Chicago on April 1950 (Soc. of Ec. Pal. and Min., spec. publ. No. 2, 1951).

Thus, although the hydrodynamical conditions for the evolution and propagation of density currents are still unclear in many details, there is no lack of evidence that their effects are of great geological importance. It may therefore be expected that further theoretical and experimental investigations, together with field observations of the fluvial processes in their natural environment, and future stratigraphic and morphological studies, will cast new light on some debatable points, e.g. microstratification in glacial varves.

The nature of turbulence. Macroturbulence

The turbulent motion in streams is of various scales, ranging from the molecular to that corresponding to the dimensions of the stream. As yet there is no concrete detailed conception of turbulence. Application of the mixing length theory has led to many important advances in research on turbulence, but the general current opinion seems to be that the basic assumptions of this theory regarding the character of the motion are not fully realistic, so that the theory is not tenable in all respects.

However, it is generally realized that eddies ordinarily originate near the bed of the river, move along with the general flow, and undergo continual changes. "The vortices when first formed may be more or less cylindrical or probably ring-shaped; however, because of the complex forces acting on such a vortex due to the mutual interaction of neighbouring vortices, it is quickly deformed into an extremely complex shape, so that its individuality is practically lost. As individual vortices move about, they may combine with others or break down into smaller ones" (KALINSKE 1943, p. 267).

In a series of experimental and theoretical investigations, BATCHELOR and TOWNSEND have endeavoured to attain a more generally valid interpretation of turbulence. A paper on the structure of the turbulent boundary layer by TOWNSEND (1951, pp. 388—389) gives the following description of the boundary layer:

"Study of the energy balance thus divides the boundary layer into an outer region of free turbulence and an inner region of turbulence similar to that in pipes and channels, distinguished by the large inward flux of energy. The inner region may be divided into the laminar sublayer, a thin dissipative layer and a much larger region where the direct influence of viscosity is negligible, and energy loss is mostly due to transfer of energy by pressure gradients."

Concerning the structure of individual eddies, TOWNSEND (p. 393) considers that "it is natural to suspect the presence of long cylindrical eddies with geometrical axis along the direction of flow, and with vorticity directed approximately along the principal axis of positive rate of strain. Such an eddy could obtain energy from the mean flow and would dissipate it partly through turbulent friction and partly by ordinary viscous forces at the wall . . . Superimposed on the system of cylindrical eddies must be a whole sequence of ordinary eddies of the more usual unorganized sort, transferring energy down the scale of eddy sizes . . . Only in the outer part of the layer is the loss of turbulent energy through the normal cascade process the dominant factor, and correspondingly, only in this part is the flow accurately in a condition of local similarity."

It is reasonable to hope that a more thorough and realistic conception of turbulent microstructure will lead to a better understanding of the morphological processes in rivers, especially the process of entrainment and the transportation of bed load.

Turbulent motion on a large scale is also important. The general term for such motion is *macroturbulence*. Description cannot be more than qualitative at present. An attempt to treat the phenomena included under this term systematically has been made by MATTHES (1947), on the basis of "observations made by the writer over a period of 15 years on natural channels ranging from brooks to rivers the size of the Lower Mississippi".

The most appropriate way to give an account of the interesting work of MATTHES seems to be to quote his classification of macroturbulent phenomena. He distinguishes between six different types of motion in accordance with the following headings:

I. Rhythmic and cyclic surges
   1. *Velocity pulsations:* inherent in all natural stream flow, affect bottom velocities more than surface velocities.
   2. *Water-turbulence fluctuations:* periodic rise and fall of water surface is shown on graphs of recording gages but is observable also on staff gages, strongest during high rising stages, decreasing in intensity during falling stages.
   3. *Surge phenomena of considerable magnitude produced by abnormal channel conditions:* local, occur at abrupt changes in direction of flow, manifested by a cycle of alternate elevation and subsidence of water surface with clocklike regularity, accompanied by eddying currents which reverse their direction of flow in some cases as a definite feature of the cycle.

II. Rotary, continuous
   1. *Slow bank eddies or rollers:* have quasi-vertical axes, occur in bays or pockets where channel has excessive width, collect floating drift, promote deposition of materials in suspension and are a cause of shoaling.
   2. *Fast bank eddies or rollers, sometimes called suction eddies:* have quasi-vertical axes, cause active bank erosion, occur at upstream as well as downstream ends of bridge abutments, bank protection works, and projecting ledge rock; erosive action often extends downward causing local bed deepening; small eddies of this general type occur at bridge piers and piling in the channel.
   3. *Slow bottom rollers:* have quasi-horizontal axes, occur during low-water stages where channel bottom has excessive depth, promote sedimentation, cease during flood stages when renewed bed deepening takes place.
   4. *Fast bottom rollers:* have quasi-horizontal axes, occur during high stages downstream from artificial or natural bed sills or low obstructions, cause bed deepening extending to considerable depths, which progresses upstream toward the sill or obstruction with tendency to undermine it.

III. Vortex action, upward, intermittent rather than periodic
   1. *Short-lived, local, upward, displacements of water entraining bottom materials, terminate at surface in nonrotating boils:* action is upward with strong and swift vortex motion at stream bed where suction lifts materials, during ascent rotary motion loses intensity and heavier materials drop out, energy of propagation suffices to raise bulk of displaced water and much of finer materials entrained to surface, boil protrudes above latter to variable heights and does not rotate, occurs along axis of main current and at any other points where sufficient energy develops, primary agency responsible for rapid bed deepening during rising stages, weak during low stages.
   2. *Vertical-axis vortices at submerged obstructions such as tall boulders, operate on upstream and downstream sides, ending in nonrotating boils at surface:* action is upward with strong and swift vortex motion at stream bed where it develops powerful suction, rotary motion loses intensity during ascent, no trace of it appearing at the surface, but energy of propagation is sufficient to carry water and bed materials so raised over the obstruction in the shape of a boil, short-lived, repeated at brief intervals, primary agency responsible for pothole formation in rock beds and along canyon walls.

IV. Vortex action, downward, sustained but subject to interruptions
   1. *Vortex axes inclined in direction of flow, trending downwards, cause erosion by fluting on submerged stationary boulders and projecting rock masses in river bed and along canyon walls:* small vortices which chisel down the corners and edges of submerged stationary rock masses during intense velocities of flood flow, produce fluting commonly seen in canyons and mountain streams, action is sustained but subject to interruptions caused by temporary changes in current direction.

V. Transverse oscillations

Superelevation of water surface at concave banks of bends: produces slope of water surface at right angles to axis of flow except at crossings where surface normally is level across the channel.

VI. Helicoidal, continuous

1. *Helicoidal flow at bends:* peculiar to deep and narrow channels, in wide and shallow streams takes on an abortive form.
2. *Double helicoidal flow in straight channels:* occurs only in long straight reaches having deep rectangular or trapezoidal cross sections, not observable in wide and shallow streams.

Several of the flow phenomena listed by MATTHES cannot be termed turbulence in the strict sense, if we take this to be a complex secondary motion with apparently random variations, superposed upon the general flow movement. But this is of minor significance, and the above classification is of great qualitative value.

Some of the flow phenomena mentioned above have earlier been described in relative detail, e.g. the eddies and rollers in group II. Thus REHBOCK (1917 and 1929) has given a detailed treatment of "*Uferwalzen*" with either vertical or inclined axis, and "*Grundwalzen*" with horizontal axis, and has discussed their morphological significance (cf. also the summary given by HJULSTRÖM 1935, p. 259).

Transverse oscillations have been considered by EXNER (1919 and 1921), among others. This type of motion has accountably played a prominent part in the discussion of meandering (HJULSTRÖM 1942).

Helicoidal flow was first described by THOMSON (1877), and subsequently by ALBERT EINSTEIN (1926), VOGEL and THOMPSON (1933), BLUE, HERBERT and LANCEFIELD (1933), and others. This type of secondary flow implies that surface water flows towards the outside of a river bend, while water near the bottom flows inward. Like transverse oscillations it has been discussed in connection with meanders. Helicoidal flow is most pronounced in deep, narrow channels, but less so in shallow, broad channels. The author has observed a slight tendency to secondary flow in the meanders of Klarälven at medium levels of the river, but has not been able to trace such flow at low river levels (p. 287).

Jet flow

The flow in a channel with diverging sides has a strong tendency to separate from the sides and form a *free jet* (p. 146). This phenomenon regularly occurs when a stream or river flows directly into a lake, a reservoir, or the sea. The flow conditions in such a free jet are of decisive importance for the transportation and deposition of material carried by the stream, and therefore for the manner in which a delta grows.

The manner in which a free jet spreads out in a liquid was first treated by TOLLMIEN (1926), on the basis of PRANDTL's mixing-length hypothesis (eq. 26). If the jet enters into a liquid where it can spread out in all directions, it forms a three-dimensional *axial jet* (runde Freistrahl). If the spreading is restricted in any direction, a two-dimensional *plane jet* (ebene Freistrahl) is formed.

When a river current flows into stationary water in a basin, it continues with a steadily decreasing velocity. At the same time an increasing amount of the surrounding water is sucked into the jet, so that the body of moving water becomes increasingly larger.

The boundaries of the jet and the decrease in its velocity away from its entrance into the basin have been discussed by BATES (1953) in connection with a new delta theory. He also gives several references to earlier work. BATES (p. 2125) distinguishes between

a. "*Hyperpycnal* inflow (inflow more dense), where the sediment-laden fluid flows down the side of the basin and then along the bottom as a turbidity (density) current. With vertical mixing inhibited during this flow because the dense fluid seeks to remain at the lowest possible level, the flow pattern is that of the plane jet. The most frequent site for delta formation by such flow is at the mouth of submarine canyons.

b. *Homopycnal* inflow (inflow equally dense), where sediment-laden fluid enters a basin filled with fluid of comparable density, as in the case of a stream entering a fresh-water lake. Mixing takes place readily in three dimensions, and the flow pattern is that of the axial jet. The type of delta which forms is the classical type with top-, fore-, and bottom-set beds as described by GILBERT (1885).

c. *Hypopycnal* inflow (inflow less dense), where sediment-laden fluid moves out over the surface of denser fluid filling the basin, as in the case of a stream discharging into the ocean. Vertical mixing is inhibited because of stability between the layers, and the flow pattern is that of the plane jet. If the magnitude of discharge is small, a lunate bar forms off the outlet; if the discharge is moderate to large, a cuspate, arcuate, or bird-foot type delta will form."

BATES describes the pattern of flow in the three cases of the above classification, and puts forward an interpretation of the development and morphology of deltas. He also discusses the effect of the Earth's rotation, winds, waves, and tides.

BATES's theory of delta formation has been criticised by CRICKMAY (1955), who points out certain terminological inconsistencies, as well as the fact that BATES has not given due attention to the geometrical form of the river estuary.

AXELSSON (1956) has shown that a difference of density between the inflowing water and the water of the basin has a greater effect on the shape of a jet in a fresh-water lake than BATES takes into account. Even small density differences affect the flow and the mixing process (cf. p. 158), and thereby influence both the decrease of the flow velocity from the mouth of the river and the spread of the current. The way in which a density current spreads out is quite different from that of a free jet.

Accordingly the theory of delta formation put forward by BATES cannot be regarded as complete. Several other factors must also be taken into consideration. However, there is no doubt that BATES has with his theory introduced a new approach to delta problems, and one that will almost certainly prove to be fruitful.

In fact, it seems that BATES's approach to the problem is the only way to reach a proper understanding of the mechanism of delta formation. There can be no doubt that the inflowing current often spreads out in the stationary mass of water according to the rules applying to a free jet. But it is also necessary to pay attention to density stratification, macroturbulence, and "meandering" (the formation of loops, cut-offs, and isolated cells of river water) in the jet. Above all, it is necessary to place the flow conditions in an intimate relation to what we know of the transportation and deposition of bed load and suspended load. Ultimately it is always the rate of sedimentation that determines the stratigraphy and morphology of a delta.

CHAPTER II

THE MORPHOLOGICAL ACTIVITY OF FLOWING WATER

EROSION OF THE STREAM BED

General

The entrainment of particles from the bed of a river and their subsequent transport by flowing water are processes that cannot be exactly described solely by equations and the corresponding physical concepts. The phenomena involved are too complex for such description. The force exerted by the flowing water on the river bed fluctuates from one instant to the next on account of turbulence; the manner in which the force acts on the particle varies with the position of the particle and the type of flow, and the resistance the particle offers to the motion depends on the position, size, and shape of the particle, as well as on whatever cohesive forces there may be between the individual particles.

The entrainment of a particle is brought about by the random cooperation of these different factors, and the particle begins to move at the instant the "active" forces first happen to exceed the "passive". If the process of erosion is thus determined by the probability that the individual particles will loosen from the substrate the description of the phenomenon becomes largely a statistical problem. Statistical considerations have in fact been successfully applied to the quantitative computation of the transport of bed-load in a water-course (cf. EINSTEIN 1950).

But nevertheless it should be stressed that the statistical computations must have a sound physical basis if they are to be successful. To understand the nature of fluviatile processes, and to be able to discuss the results of their morphological action under various conditions, it is necessary to know the fundamental physical laws governing the erosion and the transport of sediments. This is particularly true in connection with geological-stratigraphical problems, where the sedimentary environments have to be considered with the help of such measurable quantities as particle size and shape, thickness of strata and laminae, mineralogical composition, etc.

In the sequel we will first deal with criteria for erosion and transport of material under various conditions, and afterwards turn to certain morphological and geological problems, which depend for their solution on an interpretation of the erosion process. To begin with the fluid force acting on the bottom is considered, and then the resistance of particles to the fluid force on account of frictional and cohesive forces in the sediment.

The fluid force

In the technical literature it is usual to state the force exerted by flowing water on the river bed in terms of the slope of the energy grade line, the depth or the hydraulic radius, etc. The force (*boundary shear, shear stress, tractive force, force d'entrainement, Schubspannung, Schleppspannung, Schleppkraft*) can be expressed unambiguously in terms of these quantities, without the direct use of the flow velocity. The fact that the local flow velocity varies with the distance from the bottom introduces some difficulty into the definition of a flow velocity. Consequently a formula involving such a quantity will differ according to whether the flow velocity at the bottom, the mean velocity, or the surface velocity is used.

As has been particularly stressed by HJULSTRÖM (1935, p. 292; cf. also MENARD 1950, p. 149), there is no advantage from the geologist's point of view in employing the concept of tractive force. For a geologist, who is often not familiar with the hydrotechnical literature, it is more natural to make use of such terms as depth and flow velocity. Moreover, when considering the conditions under which a fluviatile sediment was deposited it is natural to discuss what the depth of water probably was at the time, and to attempt to correlate the sedimentary structure to definite flow conditions.

This difference in the approaches of the engineer and of the geologist and geographer is not a difference of principle. As has been shown in the sections on "boundary-layer thickness and resistance" (p. 142) and "the velocity distribution near a solid boundary" (p. 146), it is not difficult to obtain an expression for the tractive force as a function of the flow velocity.

According to equation (18) we have for rough flow

$$u_z = 5.75\, u_* \log_{10} \frac{30.2\, z}{k_s} \tag{40}$$

where u_z is the flow velocity at a height z above the bottom, u_* is the shearing stress velocity, and k_s is the height of the irregularities in the bottom, equal to the grain diameter in the case of a flat sandy bottom.

According to equation 6 on p. 141, $u_* = \sqrt{\frac{\tau_0}{\varrho_w}}$, where τ_0 is the magnitude of the boundary shear, and ϱ_w the density of the water. For rough flow this means that

$$\tau_0 = \varrho_w \left[\frac{u_z}{5.75 \log_{10} \left(\frac{30.2\, z}{k_s} \right)} \right]^2 \qquad \text{(rough flow) (41)}$$

Analogously, for transitional flow

$$\tau_0 = \varrho_w \left[\frac{u_z}{5.75 \log_{10} \left(\frac{30.2\, z}{\alpha \cdot k_s} \right)} \right]^2 \qquad \text{(transitional flow) (42)}$$

where α is a correction factor that depends on the relation between k_s and the thickness of the laminar sublayer. α is presented graphically in fig. 8, p. 147.

For smooth flow the shearing stress can be calculated from equations (10) and (13). The approximate formula obtained is

$$\tau_0 = 0.023\, \varrho_w\, \nu^{0.25}\, u_{\max}{}^{1.75}\, d^{-0.25} \qquad \text{(smooth flow)} \quad (43)$$

where the thickness of the turbulent boundary layer, δ_{turb}, has been taken to be equal to the depth of the water, d, and the velocity of free flow u_0 equal to the maximum velocity $u_{\max}$, which in its turn is approximately equal to the surface velocity (cf. also PRANDTL 1952, p. 128).

The boundaries for the three types of flow—rough, transitional, and smooth—are given by equations 7a—7c, p. 141.

In formulas (41)—(43) the tractive force exerted by the flowing water on the bottom is stated as a function of simple quantities such as the velocity of flow, the depth, the density of the water, and so on. It is a physically well-founded and generally accepted assumption that erosion and transportation of material sets in when this force exceeds a certain boundary value. However, it must be borne in mind that the formulas provide only the *average value* of the force. The turbulent fluctuations give rise to instantaneous values that may be considerably higher. Furthermore, no attention has been paid to irregularities in the bottom, and the shape resistance, macroturbulence, and deviations from the normal vertical distribution of velocity they give rise to.

Before these disturbing factors are discussed, however, it is necessary to clarify the resistance offered by the bottom material against erosion, for different categories of grain size and different environments.

Resistance to erosion

Frictional soils

It is customary to state the grain size of a sediment as a characteristic of its erosibility. And in fact, for coarse sediments such as boulders, gravel, and coarse sand, it has been found that the relation between grain size and erosibility is close, though not unique. Fine sediments—silt and clay—on the other hand, often deviate from this rule. Two river beds with the same distribution of grain size may nevertheless exhibit quite different resistances to erosion (cf. FORBES 1857, p. 475, quoted in HJULSTRÖM 1935, p. 299). It is therefore probable that other factors besides grain size determine the erosibility of very fine-grained material.

Properties of a sediment that might be supposed to have significance for the resistance to erosion are, besides the *grain size*, the *density* of the grains, the *particle shape*, the *porosity* of the sediment (i.e. the proportion of pore space), and the *cohesiveness* of the particles on account of surface forces. Since we first consider uniform material, we will for the time disregard the important effect that different *degrees of sorting* may have.

Soils are generally divided according to their behaviour under stress into *frictional* and *cohesive soils* (cf., for instance, CALDENIUS 1946 and JAKOBSON 1946). The boundary between the two types is diffuse: in coarse material the only forces resisting separation of the particles are the frictional, while in very fine material the cohesive forces are dominant. But for intermediate soils (grain diameter 0.006—0.6 mm approximately) both friction and cohesion are of significance; sometimes the frictional forces predominate, sometimes the cohesive, depending on porosity, water content, the mineralogical composition, etc.

Apart from the fluid force, the most important forces acting on a particle of a frictional soil on the bottom of a water-course are those of gravity and buoyancy. A particle's equilibrium becomes unstable when the resultant of these three forces happens to pass through one of the points at which the particle is supported by adjoining particles. Since the positions of such points of support can be shown to be related to the *angle of repose* of the soil (WHITE 1940, p. 324; see also fig. 11 A), the angle of repose is an important factor in estimating the erosibility of a frictional soil. The angle of repose is primarily dependent on the shape of individual particles, but also to some extent on the porosity or degree of packing. However, the variation of the angle of repose from one soil to another is not large. Extreme values of approximately 30° and 45° are mentioned in the literature; the usual value appears to be about 35°.

To sum up, the resistance of a frictional soil to erosion may be expected to depend on the immersed weight of the particles, and the angle of repose, i.e. to be describable by a function of grain size, density of the material and fluid, and the angle of repose.

Cohesive soils

The conditions in a cohesive soil are more complicated. For very fine soils the gravitational and buoyancy forces are negligibly small, while the cohesive forces are important. Cohesion resists the disruptive force due to the flow of water, and the cohesive force is therefore opposite in direction to the fluid force.

The nature of the forces binding together extreme cohesive soils, i.e. clays, is as yet incompletely understood, and an account of them cannot be more than qualitative. According to GRIM (1953, p. 9), the following forces can be distinguished:

1. "Forces due to the attraction of the mass of one clay mineral particle for the mass of another particle.

2. Intermolecular forces resulting from the nearness of one particle to another with the overlap of fields of force of molecules in the surface layers of adjacent particles.

3. Electrostatic forces due to charges on the lattice resulting from unbalanced substitution within the lattice, broken bonds on edges of the lattice, and the attractive force of certain ions adsorbed on clay-mineral surfaces . . .

4. The bonding action of adsorbed polar molecules. Oriented water molecules between two clay-mineral surfaces may form a bridge of considerable strength, if only a few molecules thick, and of no strength, if more than a few molecules thick . . ."

The binding forces in a cohesive soil determine its strength. It is therefore also reasonable to suppose that they also determine its erosibility.

In the light of the above we can hardly expect any close correlation between grain size and erosibility. As TERZAGHI and PECK (1948, p. 21) point out, "The physical properties of the clay depend to a large extent on the type of clay mineral that dominates the colloidal fraction. They also depend to a large extent on the substances that are present in the adsorbed layers. Hence, two clays with identical grain-size curves can be extremely different in every other respect. Because of these conditions, well-defined statistical relations between grain-size characteristics and significant soil properties such as the angle of internal friction have been encountered only within relatively small regions where all the soils of the same category, such as all the clays or all the sands, have a similar geological origin."

One reason that two river beds of cohesive material, and even with the same composition as regards grain size and minerals, do not exhibit the same resistance to erosion may be a difference in porosity. Whereas in a frictional soil the porosity can vary only within rather narrow limits, among cohesive soils the variation is considerable. In sandy sediments the pore space varies between 25 and 50 %, and for loosely packed sands is usually about 40 %. In clays the pore space can vary between 30 and 90 % (TERZAGHI and PECK 1948, p. 26). Shales usually have a porosity of 20 % or less (KRUMBEIN and SLOSS 1953, p. 217).

The initial porosity of a water-laden fine sediment is exceptionally high. But in due course a compaction or consolidation occurs, which is more rapid and effective if the sediment is subjected to a solid load, e.g. through the deposition of further sediment. The large decrease in the pore space enhances the cohesive forces, and the resistance to erosion becomes of course larger.

Besides the compaction brought about by the decrease in the pore space there is often a cementation of the sediment owing to the deposition of binding material in the interstices of the sediment. This mechanical or chemical process also helps to increase the rigidity of the sediment.

Thus we see that there is no simple relation between cohesiveness and grain-size. However, in general it is true that the finer the grains of a sediment the greater are the cohesive forces. This is also to be expected in view of the fact that the cohesion is a consequence of surface forces, and that the effective surface in a sediment varies inversely with the grain size if the particle shape is similar.

Although there is some connection between grain size and erosibility even for cohesive soils, it seems appropriate to make use of some other property than the grain size to characterise the erosibility of such soils. *Probably the most rational approach would be to examine the shear strength of the sediment in situ by means of some geotechnical shear test.* But until some such method, suitable as erosibility test, has been worked out and tested in the laboratory and in the field the question must be left open.

The critical erosion velocity

Definitions and earlier investigations

It has long been a matter of interest at what minimum velocity of flow sediment of a particular grain size begins to be eroded. There have been defined several different parameters of the force or current velocity necessary to initiate motion. The most common are:

critical erosion velocity or *competent velocity*, which means the slowest current velocity, ordinarily the *mean* current velocity, which is capable of initiating the movement of grains of a given diameter (MENARD 1950, p. 149), *critical tractive force*, which is the fluid force acting on the river bed, and *critical frictional velocity* or *critical shearing stress velocity*, which is a calculated quantity with the dimensions of a velocity (cf. p. 141).

Fundamental investigations concerning the critical erosion velocity were those of GILBERT (1914), SCHAFFERNAK (1922), and KRAMER (1932). HJULSTRÖM (1935) devoted particular attention to this problem. On the basis of his own observations and results obtained by several earlier investigators (PENCK 1894, GILBERT 1914, SCHOKLITSCH 1914, SCHAFFERNAK 1922, FORTIER and SCOBEY 1926, and others), he constructed a curve for erosion and deposition of uniform material. This diagram deviated in one important respect from previous comparable presentations.

"The most noticeable deviations of the erosion curve in these illustrations from older accounts, for instance, SCHAFFERNAK'S (1922, p. 14) and S. A. ANDERSEN'S (1931, p. 33), is that it has a minimum and does not go down to the origin of the coordinate system. The minimum is not at the size of particle 0 but within the range 0.1—0.5 mm. This thus indicates that loose, fine sand, for instance, of quicksand character, is the easiest to erode, whereas silty loam and clay as well as coarser sand and gravel, etc., demand greater velocities." (HJULSTRÖM 1935, p. 296.)

All the essential features of HJULSTRÖM'S diagram have been confirmed by subsequent investigations, and it is included as a standard diagram in many geological textbooks. On account of the complexity of the erosion process it is necessary to draw the diagram not as a single well-defined curve but instead as a comparatively broad band. In a later publication (1939 a, p. 27) HJULSTRÖM points out the importance of a further study of this curve: "Some of the problems that await solution are here given. 1. Determination of the velocity, or tractive force, with which material of similar or dissimilar grain size is eroded. In other words, a careful determination of curve A in Figure 1. Similar curves should be determined for different stream gradients and other hydrographic features . . ." By separating the contributions of the various factors involved it should be possible to clarify the reasons for the often pronounced differences in the critical erosion velocity, and thereby lead the way to an explanation of certain as yet unsolved morphological and stratigraphical problems.

The most fruitful approach to a closer analysis of the critical traction force or critical velocity for erosion is due to the German engineer SHIELDS (1936). By means of experiments largely carried out by himself he was able to show that *the ratio of the force exerted by the flowing water along the bottom to the resistance of a layer of sand grains is a universal function of the ratio of the grain size to the thickness of the laminar sublayer* (SHIELDS 1936, p. 12). This extremely important result makes it possible to proceed with a quantitative discussion of the effects of various factors on the critical velocity for erosion. It provides the theoretical foundation on which the subsequent presentation here is based.

One of the first detailed investigations of the equilibrium conditions for sand grains on the bed of a stream was carried out by WHITE (1940). More recent American work, in particular that of KALINSKE (1947), has extended WHITE's results, and utilised them as a basis of calculations of the quantitative transport of bed-load.

The mechanism of entrainment in frictional soils

In principle the loosening of an individual sand grain and the transport of it over the bottom may be regarded as follows. The bottom exerts a drag on the flowing water. The corresponding tangential force which the stream exerts on the bottom affects first of all the uppermost grains of the bed. The types of forces acting on a grain of sand are indicated in fig. 11, where diagrams A and B are essentially from WHITE (1940. p, 324).

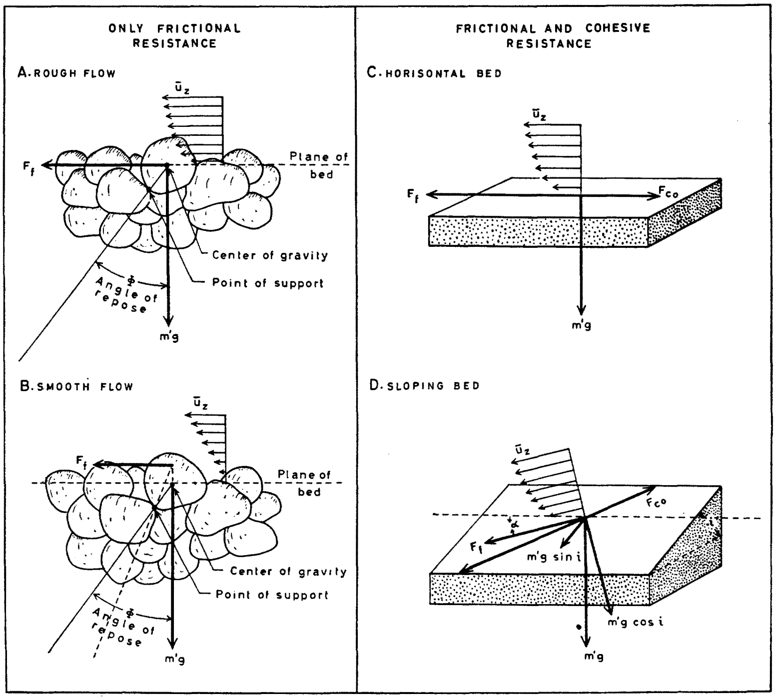

Fig. 11. Forces acting on grains in a river bed. A. Equilibrium of a grain when the flow is rough, assuming that the "lift" component is negligibly small. B. Equilibrium of a grain when the flow is smooth. C. Equilibrium of a grain on a horizontal bed when frictional forces as well as cohesive are acting. D. Equilibrium of a grain on a sloping bed with frictional and cohesive forces. (A and B redrawn from WHITE 1940, p. 324.)

For relatively high flow velocities and large grains the flow is rough. There occur pressure differences between the upstream and downstream sides of each grain, and small eddies are shed from the grains. On each particle there is thus a resultant force which, for spherical particles, may be taken to pass through the centre of gravity. This force is not constant: it varies with the turbulent fluctuations of velocity in the main stream, and with the shedding of the small eddies. WHITE (p. 333) has shown that the hydrodynamic lift or vertical force is quite insignificant in the case of bed-load movement, and that the resultant fluid force may therefore be regarded as acting parallel to the bottom.

Fig. 11 A is a schematic diagram of the forces acting on a sand grain. F_f is the instantaneous horizontal fluid force, and $m'g$ is the algebraic sum of the gravitational and buoyancy forces. It is easy to see that the grain can begin to move when $F_f = m'g \tan \Phi$, where m' is the immersed weight of the grain, g the acceleration due to gravity, and Φ the angle of repose.

It is obvious that F_f is a function of the boundary shear τ_0. If the particles are assumed to be spherical, each grain has effectively a surface $\frac{\pi k^2}{4}$ that may be subject to this stress (k is the particle diameter). The instantaneous critical boundary shear, τ_c, is then $\frac{4 F_f \cdot \gamma_1}{\pi \cdot k^2}$, where γ_1 is a factor which is simply related to the degree of packing or the porosity of the sediment. If movement commences when the average boundary shear τ_0 attains a value $\gamma_2 \tau_c$, the final formula for the initiation of movement may be written $\left(m'g = \frac{\pi k^3 (\varrho_m - \varrho_w) g}{6} \right)$

$$\tau_0 = \frac{2 \gamma_1 \gamma_2 k (\varrho_m - \varrho_w) g}{3} \tan \Phi \tag{44}$$

In equation (44) γ_1 is a factor that depends only on the distribution of sand grains on the bottom. For uniform material the variation of γ_1 is very small. In WHITE's experiments the value of γ_1 is about 0.35. γ_2 depends mainly on the degree of turbulence, and will be discussed later (p. 175).

In the case illustrated in fig. 11 B, all the grains are in the laminar sublayer. The force acting on a grain is then due entirely to the viscous stresses. These act parallel to the bottom, like the forces in case A, but the resultant force acting on a grain now acts along a line passing above the centre of gravity of the grain. As in the preceding case, the force fluctuates, since the velocity fluctuations in the turbulent main flow affect the flow even in the sublayer.

An equation may be derived for case B in a manner analogous to that used for equation (44). However, as WHITE has pointed out, it is impossible to deduce the position of the line along which the viscous force acts. WHITE solved the difficulty by inserting in equation (44) a further factor, which can then be determined experimentally. But in view of SHIELDS's results it seems more consistent to overcome the difficulty by including this factor in γ_2. γ_2 then becomes a coefficient with a value depending on the ratio of the grain size to the thickness of the laminar sublayer. It is proportional to the function given by SHIELDS, and mentioned here on p. 176. Equation (44) is thus rendered valid even for the case where the grains all lie within the laminar sublayer.

The mechanism of entrainment in cohesive soils

Cases A and B refer to a river bed entirely composed of frictional material, where cohesive forces are altogether absent. The situation when cohesive forces affect the resistance to erosion is indicated in fig. 11 C. The limiting value of the cohesive force on an individual grain is represented by the force F_{Co}. At the present state of our knowledge it is not possible to state F_{Co} as a function of the particle size or some other variables, but, as already pointed out, it is probable that it could be correlated to the shear strength of the sediment.

The movement of a single particle in this case should commence when $(F_f - F_{Co})$ attains the value $m'g \tan \Phi$. It is easily shown that this leads to the formula

$$\tau_0 = \gamma_1 \gamma_2 \left[\frac{2k\,(\varrho_m - \varrho_w)\,g}{3} \tan \Phi + \gamma_3 C \right] \tag{45}$$

where γ_3 is a factor of proportionality, and C is the cohesion of the material (cf. TERZAGHI and PECK 1948, p. 82).

Formula (45) ought to be valid for any material. For frictional soils the second term in the bracket is negligible, and the formula becomes the same as (44), while for purely cohesive soils the first term becomes small, and for intermediate material both terms are of the same order of magnitude.

In this connection it should be stressed that, according to the author's own experience, consolidated fine sediment is often eroded not particle by particle but in aggregates of particles (cf. p. 279). These aggregates vary in size, sometimes attaining centimetre or even decimetre size, in which case weak zones along bedding planes or cracks and surfaces of sliding allow the lumps of clay to break away. It is also likely that corrasion by coarser particles, sand or gravel, which the flowing water sometimes carries with it along the bottom also plays an important part in the erosion of fine sediment.

The entrainment process on sloping surfaces

From the morphological point of view an important case is that where erosion or transport of sediment takes place along an inclined surface. Instances where this case finds application are in the question of lateral erosion, or the transport of material up the proximal slope or down the distal slope of ripples or banks.

A diagrammatic representation of the situation is given in fig. 11 D. We suppose that the surface in question is inclined at an angle i to the horizontal. In the general case the water flows down the inclined surface in a mean direction at an angle α with a horizontal line in the surface. The immersed weight of the particle $m'g$, may be resolved into components $m'g \cos i$ perpendicular to the surface and $m'g \sin i$ in the direction of greatest slope. The fluid force F_f and the force component $m'g \sin i$ combined give the resultant force tending to loosen the particle, while the cohesive force F_{Co} and the gravitational component $m'g \cos i$ resist this tendency.

The most interesting cases from the morphological point of view occur when α is zero, i.e. when the flow is parallel to the inclined surface (lateral erosion), and when $\alpha = \pm 90°$, i.e. when the flow is directly up or down the inclined surface. It is not difficult to show that in the former case the formula obtained is

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$$\tau_0 = \gamma_1 \gamma_2 \sqrt{\frac{4k^2(\varrho_m - \varrho_w)g^2}{9}\left(\tan^2 \Phi \cos^2 i - \sin^2 i\right) + \frac{4\gamma_3 k(\varrho_m - \varrho_w)gC}{3}\tan \Phi \cos i + \gamma_3{}^2C^2} \quad (46)$$

where the symbols have the same significance as previously. In the latter cases we get

$$\tau_0 = \gamma_1 \gamma_2 \left[\frac{2k(\varrho_m - \varrho_w)g}{3}\left(\tan \Phi \cos i \pm \sin i\right) + \gamma_3 C\right] \quad (47)$$

where the plus sign refers to flow up the inclined surface, and the minus sign to flow down the slope.

Formulas (46) and (47) give the limiting values of the average tractive force along the bottom for erosion in various soils. For frictional soils C is negligible. Where the inclination of the river bed is zero, both (46) and (47) become identical with (44). On the other hand, if the frictional and cohesive forces are of the same order of magnitude and the inclination is zero, both formulas become identical with (45). Hence (44) and (45) are special cases of the more general formulas (46) and (47).

It is obvious that comprehensive laboratory experiments would be needed to obtain definite verification of the formulas, and in particular to examine the two parameters γ_3 and C. Unfortunately, it has not been possible to carry out the requisite experiments within the framework of the present work. But the author intends to return to the matter if possible in a subsequent publication.

Even at the present stage, however, it is possible to draw certain general conclusions that are of morphological and geological interest. These are in the main reserved until the next section, though two comments may be made here. It follows from equations (46) and (47) that for a purely frictional sediment ($C = 0$) the tangential fluid force required for erosion to commence decreases as the inclination of the bottom increases when the flow is not up the slope. The slope of the subaquatic declivities in a water-course must be *less* than the angle of repose for the material in question if erosion is not to occur. It will also be seen that increase of the density of a frictional sediment raises the limiting value of the fluid force necessary for erosion. For a purely cohesive material, on the other hand, neither slope nor density have any effect on the critical velocity for erosion.

The critical velocity equation and the critical velocity curve

The aim of the above discussion was to analyse the manner in which the critical velocity depends on the grain size of a uniform sediment. On the basis of the conclusions arrived at it is possible to make a quantitative assessment of other factors such as density, depth of water, etc. Let us consider the simplest case, where the eroded surface is horizontal; this is the only case where it is possible to obtain verification from previous laboratory experiments.

Equation (45) may be combined with one of the equations (41)—(43). We choose (42), since the correction factor α there can be assigned values such that the equation is valid even for rough flow and to some extent even for smooth flow (cf. fig. 8, p. 147). The equation obtained is

$$u_{z_{\mathrm{Crit}}} = 5{\cdot}75 \log_{10}\left(\frac{30.2\, z}{\alpha \cdot k_s}\right) \sqrt{\frac{\gamma_1 \gamma_2}{\varrho_w}\left[\frac{2k\,(\varrho_m - \varrho_w)\,g}{3}\tan\Phi + \gamma_3 C\right]} \tag{48}$$

For rough flow and frictional soils ($\alpha = 1$, $C = 0$), (48) becomes

$$u_{z_{\mathrm{Crit}}} = 5{\cdot}75 \log_{10}\left(\frac{30.2\, z}{k_s}\right) \sqrt{\frac{2\gamma_1 \gamma_2 k\,(\varrho_m - \varrho_w)\,g}{3\,\varrho_w}\tan\Phi} \tag{49}$$

Let us first discuss equation (49). For uniform sediment with grains of diameter k (i.e. $k_s = k$), without ripples or banks, all the quantities involved are easily determined without laboratory experiments, with the exception of the coefficients γ_1 and γ_2. As already mentioned, the variation in γ_1 is very small, so that it may be regarded approximately as a constant for a uniform river bed. It remains therefore to determine γ_2.

γ_2 is a function of the ratio of the average boundary shear τ_0 to the instantaneous critical boundary shear τ_c. Consequently, it is primarily the magnitude of the velocity fluctuations near the bottom that determines the value of γ_2. In the section on "velocity fluctuations" (p. 151) it was mentioned that, according to KALINSKE (1943), "fluctuations equal to twice the mean velocity at a point in the turbulent zone near a boundary can readily be expected". Since the shear stress varies as the square of the velocity, it might be expected that γ_2 for fully turbulent flow should be of the order of 0.25.

But γ_2 must also depend on the manner in which the sand grains on the bottom are subject to the fluid force. SHIELDS (1936, p. 11) succeeded in establishing that γ_2 is a direct function of the ratio of grain size to the thickness of the laminar sublayer.[1]

On the basis of equation (49), and utilising the values of γ_2 that can be derived from SHIELD's results (see fig. 12), it is now possible to construct curves representing the relation between particle size and the critical velocity for erosion of frictional sediment. It may be observed that the critical velocity is proportional to $\sqrt{\frac{\varrho_m - \varrho_w}{\varrho_w}}$, and to $\sqrt{\tan\Phi}$, as well as almost proportional to $\sqrt{k}$ (though not exactly, since k also affects the vertical velocity distribution and the quantity γ_2). Furthermore, the numerical value of the limiting velocity depends on the height z above the bottom at which the velocity is measured.

Apart from k, the variables of particular interest are the depth of the water and the density of the sediment concerned. In principle the critical velocity can be determined for any z-value. But for practical reasons it is simplest to choose the *maximum velocity* or the *surface velocity*, which means that z is in general very nearly the same as the depth

[1] SHIELDS did not consider the quantity γ_2. He has instead a function which in the present notation is $\frac{2\gamma_1\gamma_2\tan\Phi}{3}$. Thus SHIELDS's result actually assumes that γ_1 and Φ are constants. If one puts $\gamma_1 = 0.35$, $\gamma_2 = 0.25$, and $\Phi = 35°$, then $\frac{2\gamma_1\gamma_2\tan\Phi}{3} = 0.04$ which is in good agreement with SHIELDS's result. For, SHIELDS gives the maximum value of the fun ction as 0.06 for fully rough flow, and a minimum value of 0.03 for transitional flow.

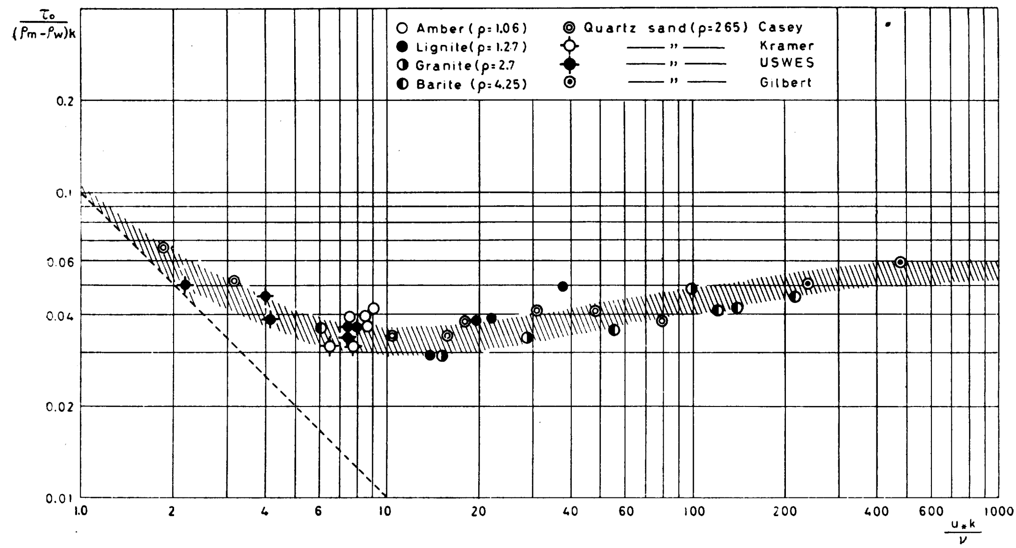

Fig. 12. The entrainment function of SHIELDS (1936, p. 11). $\frac{u_* k}{\nu} \sim \frac{5\,k}{\delta_{\text{sub}}}$, where k is the grain diameter and δ_{sub} the thickness of the laminar sublayer.

of the water d (the maximum velocity usually occurs immediately below the surface of the water, unless the normal flow velocity distribution is disturbed by macroturbulence or density stratification).

The curves are drawn for different values of z: 0.01, 0.1, 1.0, and 10 metres, which implies that in practice the water-course has these depths. It is assumed that γ_1, ϱ_m, and Φ are constants. The density of the sediment is taken to be $\varrho_m = 2.65\ g\,cm^{-3}$, which applies to the commonest material in a river-bed, quartz sand. The four curves are reproduced in fig. 13.

The various types of flow—rough, transitional, and smooth—are also indicated in the diagrams. The boundaries between them have been calculated from formulas (7 a)—(7 c) on p. 141.[1] It is in the nature of turbulent flow that these boundaries are very diffuse. Actually, the boundaries vary somewhat with the depth of the water, but this variation is small, and in view of the inevitable diffuseness of the boundaries it has been disregarded.

Critical wind velocity for eolian erosion

Curves for eolian erosion have been included in the diagram for comparison. These are based on formula (45), like those for fluviatile erosion. The tractive force of the wind has been computed for three states of stability in the air layer nearest the ground: stable

[1] As already pointed out (p. 141), these boundaries are diffuse, and have been stated differently by different authors. For instance, WHITE (1940, p. 324) and INMAN (1949, p. 55) give a definite boundary between rough and smooth flow at $\frac{u_* k_s}{\nu} = 3.5$ without a transitional zone. However, both the original experiments of NIKURADSE (1933) and later laboratory investigations indicate beyond doubt that a transitional region exists.

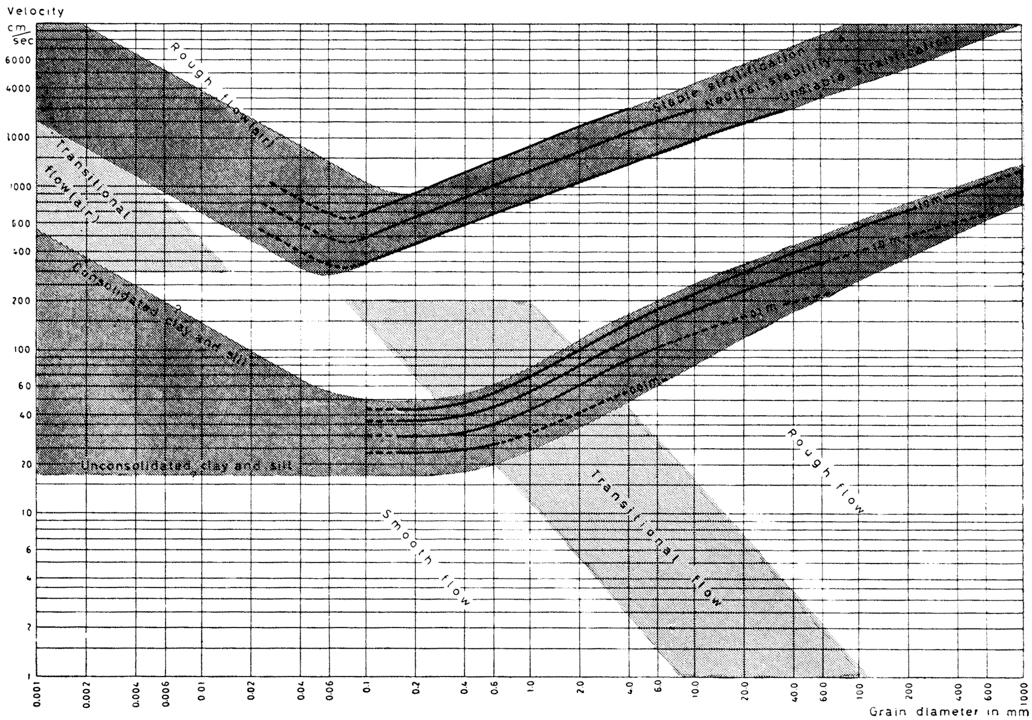

Fig. 13. Curves showing the relation between particle size and critical erosion velocity for uniform material of density 2.65 g cm^{-3}. The flow velocity is that at 0.01, 0.1, 1.0, or 10.0 m above the river bed. The lower dark band indicates supposed boundaries for variations in the critical erosion velocity. It must be stressed that the boundaries for cohesive soils are very uncertain, and that loose, unconsolidated clay or silt may be entrained at flow velocities considerably lower than the boundary values indicated. Curves for the critical erosion velocity for eolian erosion are included in the upper part of the diagram.

stratification, neutral stability, and unstable stratification. The velocity refers to 1 metre above the ground. Since the greater part of the eolian curves falls within the region for rough flow, it has been assumed that γ_2 is constant. Its value has been determined from the results of BAGNOLD (1941). The value of $\frac{2\gamma_1\gamma_2 \tan\Phi}{3}$ is approximately 0.02, which seems plausible if one considers that the velocity fluctuations in the air layer nearest the ground are large compared with the corresponding fluctuations in a water-course. The shape of the curve for neutral stability agrees very well with the curve induced empirically by BAGNOLD. For a discussion of eolian processes reference may be made to BAGNOLD (1941) and SUNDBORG (1955).

The cohesiveness and the critical velocity curve

Before commencing a more detailed discussion of the curves for water erosion, the transition to fine, cohesive sediment will first be briefly considered.

As may be seen from fig. 12, which is reproduced from SHIELDS, the function $\frac{\tau_0}{(\varrho_m - \varrho_w)\,k}$

(and consequently γ_2 also) approaches a line as k/δ_{sub} decreases (δ_{sub} is the thickness of the laminar sublayer), a line which is at 45° to the coordinate axes. This implies that for smooth flow γ_2 should be inversely proportional to k/δ_{sub}, and that $u_{z_{crit}}$ in equation (49) should approach a constant value. The curves in fig. 13 should accordingly have a *horizontal continuation* in the region for smooth flow. However, this is quite at variance with the general conclusions reached above concerning the forces acting in erosion. But SHIELDS's curve for small k/δ_{sub} is based on extrapolation from a few values for fine sediments, as may be seen from the figure.

WHITE carried out a large number of erosion experiments with water and with a more viscous oil, and he found for small values of $\frac{k}{\delta_{sub}}$ that γ_2 is practically constant. This result implies that the critical velocity $u_{z_{crit}}$ should become *smaller* as the particle size decreases. WHITE's results seem to be free from objection, but it should be observed that he made k/δ_{sub} small not by decreasing k but by increasing δ_{sub} with the help of a viscous liquid. Thus WHITE did not investigate cohesive material.

Our problem can be stated as follows: If only the frictional resistance is taken into account the critical velocity should decrease as the bottom sediment becomes finer. But all experience (cf. HJULSTRÖM 1935) indicates that the critical velocity rises for the finest grades of material. We must therefore assume that cohesive forces are involved, and we take these into account by means of equation (48). We may ask, at what particle size do the cohesive forces begin to become appreciable, and how do they vary as the particle size diminishes? In other words, what is the minimum velocity for erosion, and how does the critical erosion velocity vary as the particle size diminishes?

We must first observe that, even if we disregard the doubtful extrapolated part of SHIELDS's curve in fig. 12, the variations in γ_2 on account of the transition from rough flow via transitional flow to smooth flow should lead to a flattening of the curve in fig. 13, and a tendency to a minimum roughly at the boundary between transitional flow and smooth flow. Since this boundary is in the interval of particle size 0.2—0.5 mm, the position of the tendency to a minimum coincides exactly with the minimum value observed by HJULSTRÖM. However, if the cohesive forces did not come into the picture the curves would once more slope downwards as the particle size decreases, corresponding to a tractive fluid force according to equation (43).

It is a general observation that recently deposited, very loose and unconsolidated clay or silt may be swept away by quite a small change in the flow velocity. BATES (1953, pl. 2142), for instance, states that the very loose, flocculated ooze in the Mississippi delta is transported seaward as soon as the water level rises a few feet. Similar observations have been made by the author in Klarälven, and by L. ARNBORG in the estuary of Ångermanälven on the Gulf of Bothnia, according to information from him.

It has been mentioned previously that it is not possible at the present state of our knowledge to give an explicit expression for the magnitude of the cohesive forces, although it seems that it varies inversely as the particle size, if all other factors remain unchanged.

In this connection it may be remarked that the latter supposition coupled with equation (43), yields a slope of the curve in the region for fine material that agrees both with HJULSTRÖM's curve for fluviatile erosion and with BAGNOLD's curve for eolian erosion. In view of our very imperfect knowledge of the physical processes involved, and the wide variation in field observations, the correct procedure seems to be not to complete the curves for the finest sediments, but instead to designate the critical velocity for erosion by a broad band. The lower edge of this band has arbitrarily been drawn horizontally, while the upper edge has the slope $^4/_7$, in accordance with equation (43). However, it may be stated quite definitely that there is generally a minimum of the erosion velocity within the interval 0.2—0.5 mm of particle size, and thus an increase of the critical velocity for erosion both towards finer and towards coarser sediments, in conformity with HJULSTRÖM's curve.

In the case of eolian erosion there are two factors that contribute to the somewhat different shape of the curve, compared with the fluviatile case. The cohesive (and adhesive) forces assume significance before the flow has changed from the rough to the transitional type. Hence corresponding to the broad minimum in the curve for water there is a much more marked minimum in the curve for wind erosion. Moreover, this minimum is for dry soils displaced somewhat to the left. In air $(\varrho_m—\varrho_w)$ is greater than in water, cf. equation (48), and the predominance of the frictional forces therefore continues towards slightly finer soils than is the case for water. However, in the case of wind erosion capillary forces in the soil are often important, and the minimum may therefore be expected to be displaced somewhat to the right for moist soils. The dotted parts of the curves in the figure indicate values that have been computed on the basis of BAGNOLD's results, while the shaded band indicate supposed limits of variation for different degrees of moistness in fine soils.

Synopsis of the critical erosion curve

To sum up, the curve for fluviatile erosion may be described as follows.[1]

It is possible to distinguish four different regions or intervals between which the process of entrainment differs to some extent.

1. For the coarsest bottom material the flow is completely rough, and the turbulence extends deep into the interstices between the surface grains. In this region γ_2 is constant.[2] The curve is smooth and slightly concave downwards. This region extends to grain diameters of 6—8 mm (the figures mentioned here refer to a depth of 1 metre and material of density 2.65; for other depths and other material the boundaries are somewhat displaced).

2. The main flow is still rough for finer material, i.e. the bottom has the same resistance to flow as a completely rough surface. However, this resistance is not distributed over all the particles of the uppermost layer as it was in the preceding velocity interval. The tur-

[1] The description is partly based on the corresponding description by SHIELDS (1936, pp. 12—13) of fig. 12.

[2] For *very high* velocities and very coarse material the situation is uncertain. It is possible that the value of γ_2 may decrease.

bulence does not extend so far down among the particles. Grains in more shielded positions are not reached by the turbulent flow and therefore do not offer any shape resistance. The force acting on the most exposed grains is so much the greater, which explains the decrease of γ_2 and the downward trend of the erosion velocity in fig. 13. This interval extends from 6—8 mm down to 2 mm grain diameter.

3. For grain diameters less than 2 mm the main flow is no longer completely rough, but can be characterised as transitional. The previous tendency of the surface resistance to become concentrated on the most exposed grains is intensified, and gives rise to a minimum of γ_2 at an approximate grain diameter of 0.8—1.0 mm. For smaller particles γ_2 again increases, and at a particle diameter of 0.3 mm the grains are completely enclosed in the laminar sublayer, i.e. the main flow is completely smooth. The increase in γ_2 implies a tendency to a minimum in the erosion velocity in the interval of particle diameters 0.2—0.5 mm.

4. The form of the curve for the critical velocity for erosion below 0.3 mm grain diameter in uniform material depends on the magnitude of the cohesive forces. The flow velocity needed for erosion increases for consolidated fine sediment. For very loose silt or clay that has recently been deposited it may happen that the erosion velocity decreases as the particle size decreases. The conditions in this interval are still incompletely known.

Comparison with experimental results

Introduction

The formulas and curves for the critical erosion velocity have been partly deduced by an analysis of the probable process of entrainment for an individual grain. The proportionality factors, γ_1, γ_2, and γ_3, introduced and defined in this deduction have then been evaluated with the help of previous laboratory investigations. It is a matter of great interest to obtain a reliable estimate of the *accuracy* with which the actual critical velocity can be determined with the aid of the formulas or the curves. We will therefore make a comparison of the calculated values of the critical erosion velocity and measured values.

Anyone who has studied erosion processes or the transport of bed load knows how difficult it is to obtain exact values of many of the quantities to be measured. The motion of the particles does not begin instantaneously as soon as the flow velocity has attained a particular value. What happens is that individual grains first begin to oscillate, and then in an apparently random manner are dragged along by the current; as the flow velocity further increases this movement extends to more and more grains, until gradually there develops what may be called a general transport of material. If the flow velocity is subsequently allowed to decrease the motion continues even at velocities that are lower than that which just sufficed to initiate the motion.

Thus there is a pronounced difference between the velocity required for *initiation* of the movement and that for *cessation*. How should the critical erosion velocity then be defined? This and other questions are best approached by first clarifying the concepts involved.

Initiation of the movement

KRAMER (1932, p. 13) has defined four rates of bed load movement for successively increasing flow velocities:

1. *"Keine"* (*None*) transport of bed load signifies a condition where no grains are in motion.

2. *"Schwache"* (*Slight*) transport signifies that some small countable number of the smallest grains are set in motion at various positions on the bottom.

3. *"Mittlere"* (*Moderate*) transport signifies that medium-sized grains are set in motion everywhere on the bottom, to such an extent that the moving grains cannot be counted.

4. *"Allgemeine"* (*General*) transport signifies a motion which includes grains of all the sizes occurring, even the largest, and which leads to a relatively rapid change in the appearance of the bottom.

CASEY (1935, p. 34), and the United States Waterways Experimental Station (USWES 1935, p. 16) used more or less the same classification as KRAMER, and like him defined the critical tractive force as "that tractive force which brings about general movement of bed load mixture".

However, it was found to be so difficult to obtain a reliable classification of the motion by ocular means that both CASEY (p. 35) and USWES (p. 33), independently of one another, introduced quantitative criteria for the rate of transport to be regarded as general. Their respective values were 12.5 cm^3/min./meter width of the bottom and 14.0 cm^3/min./meter.

SHIELDS (1936, p. 9) determined the critical tractive force by extrapolating the experimental curve for the amount of transported material to the point where the transport was zero. The rate of movement thus determined conformed most closely with KRAMER'S "slight movement".

The various authors who have investigated the critical velocity or the critical tractive force have thus defined the initiation of movement somewhat differently. Moreover, the element of subjectivity in judging the state of the movement is always present, and therefore we cannot expect absolute agreement between different experimental results. In the present publication the results of previous experimental investigations have been adjusted to correspond to a *critical erosion velocity which is the velocity at which the moderate rate of movement is attained.*

Cessation of the movement

SCHAFFERNAK (1922, p. 13) observed that sediment, once it has been set in motion, continues to move even if the flow velocity falls below the critical erosion value. He found that the cessation of bed movement occurred at a flow velocity, *"the lowest transportation velocity"* or *"the deposition velocity"*, which is about two thirds of the critical erosion velocity.

This result has been discussed by HJULSTRÖM (1935, pp. 320 ff.), who showed that the difference in the velocity for erosion and deposition is of great importance when interpreting the stratigraphic composition of a deposit.

SCHAFFERNAK carried out experiments only for grain sizes greater than 5 mm. MENARD (1950, p. 151) has extended the investigations to monodisperse sands down to a grain size of 0.06 mm in flume tests at the Woods Hole Oceanographic Institution. He states: "The sands tested all came to rest if the mean current velocity was decreased to about two-thirds of the competent velocity. Even the very fine sand was deposited, although the mean horizontal current velocity was more than ten cm/sec."

In the curves of fig. 13 only the critical erosion velocity is included, and *not* the velocity for cessation of the movement.

The effect of ripple formation on the critical erosion velocity

If the bed of a watercourse is composed of sand of grain size approximately 0.2—1.0 mm, *current ripples* or *ripple marks* form soon after bottom transport is initiated. This type of transport is described on p. 207. In the present connection the important point is that the flow velocity required to initiate movement on a rippled bed is not the same as that required in the case of a smooth bed.

MENARD (1950, p. 159) expresses this fact as follows:

"Sand grains can be moved by a slower current if the bed is rippled than if it is smooth, but the difference amounts to only a few centimetres per second.

Ripples usually are formed if fine-grained sands move at all; yet, if the grains are coarser, the rippling velocity may be larger than the competent velocity. In the latter instance, the movement of sediment at low velocities occurs in a smooth phase."

The fact that sand grains on a rippled bed are more easily set in motion than grains on a smooth bottom may be explained qualitatively as follows.

The quantity k_s in eq. (49) on p. 175 represents the equivalent sand roughness. For a smooth surface this is practically the same as the grain size k. For an uneven surface with ripples or bars the value of k_s increases with the magnitude of the irregularities. It may be seen from the formula that this implies a lower critical erosion velocity.

However, it should be observed that the formation of ripples leads to a shape resistance in addition to the ordinary surface resistance (cf. p. 145). Therefore, not the whole of the flow resistance is associated with the individual grains, when the bottom is uneven, but only the actual frictional resistance (cf. EINSTEIN 1950, p. 90).

The separation of the flow on the steep downstream sides of the ripples entails a concentration of the surface resistance to the upstream side of each ripple, especially to the places where the effect of the current is strongest, i.e. the middle and upper parts of the upstream side of each ripple (cf. fig. 7, p. 145). Sand grains first begin to move at these places, at a flow velocity somewhat lower than that required for the first movement on a smooth bed.

Briefly one can say that the lowering of the critical flow velocity for bed load movement when the bed is rippled may be accounted for by the change in the state of turbulence, and thereby in the vertical velocity distribution, caused by irregularities in the bed, together with a concentration of the frictional resistance to particular regions of the rippled surface.

The formulas that have previously been derived for the critical erosion velocity apply to an even bed. Accordingly, it is necessary to disregard experiments where the sand surface was rippled when comparing the formulas with laboratory results.

Some other factors affecting experiments

The formulas and curves are valid for uniform bed material. When the bed material is instead a mixture of different grain size fractions — unsorted material — some complication is introduced. The erosion and transport of unsorted material will be dealt with later (p. 185). In comparing calculated and measured values of the critical erosion velocity only results for uniform or well-sorted bed material have been included.

Furthermore, it should be mentioned that the comparison does not apply to flowing water with a high content of suspended material, nor to a bed composed of material with markedly unsymmetrical grains (e.g. detrital mica flakes). These factors are taken into consideration in a subsequent section.

Finally, it must be pointed out that perhaps the greatest obstacle to an exact treatment of previous flume experiments is the variation in the manner in which the experiments have been carried out and the results stated. In some cases the mean flow velocity is given, in some the surface velocity, in others the critical shearing stress velocity or the critical tractive force. Sometimes the depth of water is stated, but more usually it is not.

The formulas derived here require a knowledge of the depth and the surface velocity or the velocity at a given depth. So observations where these quantities have not been measured or stated have as a rule been disregarded, unless it has been possible to compute the required values indirectly. In some cases, for instance, the surface velocity has been calculated from values of the mean velocity, the depth, and the grain size, with the aid of an assumed logarithmic distribution of velocities.

Comparison between calculated and measured values of the critical erosion velocity

Fig. 14 shows the degree of agreement between calculated and measured values of the critical erosion velocity. The calculated values have been obtained with the help of the curves in fig. 13, taking into account the depth of water, as well as the flow velocity and the grain size and density of the bed material. The calculated velocities have been plotted against those determined experimentally.

The comparison utilises results obtained by GILBERT (1914), KRAMER (1932), CASEY (1935), USWES (1935), MAVIS, HO, and TU (1935), MAVIS, LIU, and SOUCEK (1937), and NEVIN (1946).

The spread of the points in the diagram may appear surprisingly great. The main reason for this appears to be the difficulty in defining and determining the commencement of the erosion and transport process unambiguously. It may be seen, for instance, that the values of HO and TU, and those of NEVIN lie quite closely along two lines parallel to the 45° line marked in the diagram. The values of HO and TU lie to the left of the marked line, and NEVIN's to the right of it. HO and TU probably determined the velocity at which

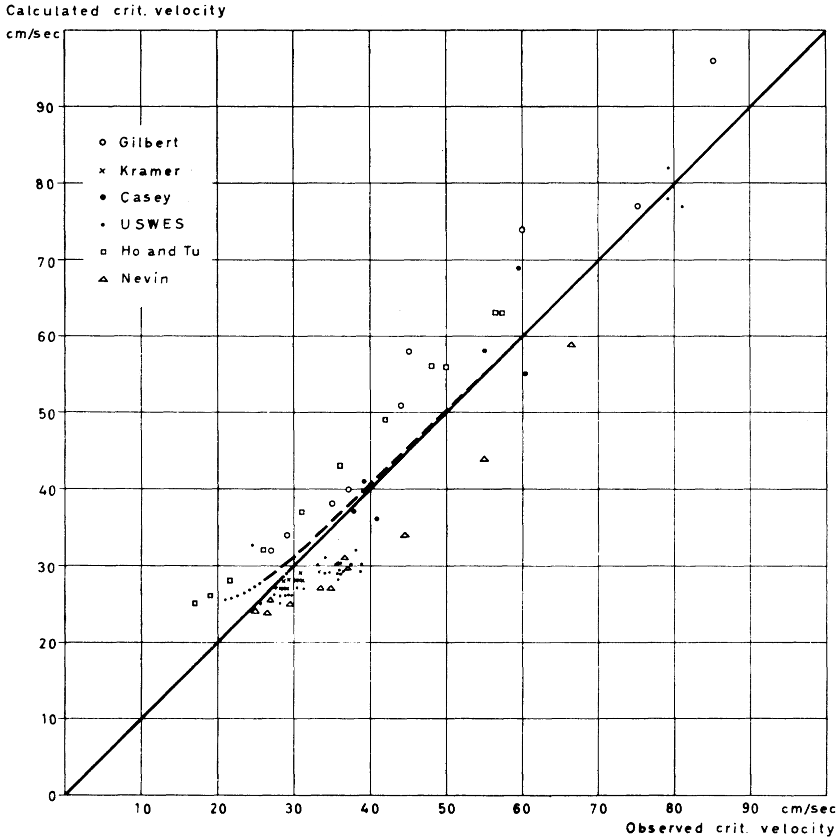

Fig. 14. Comparison between calculated and measured values of the critical erosion velocity. The dashed curve illustrates the agreement with values derived from the curve constructed by MENARD (cf. the text).

individual grains began to move, while NEVIN required the transport to be more general.

These and other difficulties make it unlikely that the agreement could be better than that actually obtained. It seems justifiable, however, to claim that the observations confirm the correctness of the premises employed in deducing the curves. For velocities greater than 100 cm/sec. there are unfortunately no data that are directly usable for the present purpose, though the results obtained by SCHAFFERNAK (1922, p. 14) for grain sizes up to 70 mm and velocities up to 200 cm/sec. fit the calculated values well.

The diagram also contains a dashed curve, which is intended to illustrate the relation between the erosion velocities calculated from the author's curves and those derived from the erosion curve constructed by MENARD (1950, p. 150). MENARD's curve refers to the mean velocity and to an unspecified depth of water ("only to currents less than one meter deep"). For purposes of comparison it is necessary to assume some particular relation between the mean velocity and the velocity at a given depth, and furthermore to assume a certain normal depth for the experiments. Such a procedure is always liable to introduce difficulties. It is accordingly possible that the relations used for the calculations (that the mean velocity is 85 % of the surface velocity, and that the normal depth for the experiments was 4 cm) does not entirely conform to the actual conditions.

The agreement between MENARD's curve and that of the author seems to be satisfactory with the possible exception of the lowest velocities (i.e. the smallest grain sizes). The discrepancy at low velocities can perhaps be attributed to the circumstance that in laboratory experiments the depth of water is usually smaller for the smaller grain sizes. It is also likely that the degree of sorting of the bed material is of great importance for just these small grain sizes.

In conclusion it may be stressed that new laboratory experiments would be very valuable in order to study the behaviour of the variables in formula (49) more closely.

Sediment mixtures

Only material of uniform grain size has been discussed in preceding sections. The derivation of the erosion curves in fig. 13 assumed that variations in grain size could be neglected. But in a natural watercourse sediments are not uniform, neither in shape, nor in size, but actually comprise a more or less heterogeneous mixture of different types of grain. The different grain size fractions in a sediment mixture are not all affected in the same way by flowing water: one or more of the fractions is more easily set in motion than the others. All the processes which entail mechanical sorting of grains with respect to size, shape, density, etc., may be referred to the selectivity of erosion, transport, or deposition. We may then enquire how a sediment mixture reacts to the forces exerted by the flowing water in comparison with a uni-granular material.

According to GILBERT (1914, pp. 184—185) the amount of material transported changes if a previously uniform sediment is mixed with material of another grain size. "It is especially notable that when fine material is added to a previously homogeneous material not only is the total capacity increased, but the capacity for the coarser part of the load is increased, and it may even be enlarged several fold. The general effect of adding coarse to fine is to reduce the stream's capacity for the fine, but under some conditions there is a slight increase."

SCHAFFERNAK (1922, p. 20) observed that at a flow velocity lower than that required for the transport of all grain sizes present the *finer* fractions of a sediment could move by themselves, leaving the coarser behind to form in due course a bed of erosion-resistant material. He used for his experiments material of grain size 5—70 mm.

On the other hand, it is stated from the investigations of USWES (1935, p. 32) that: "This somewhat paradoxical condition, in which the material first moved by the flowing water was *larger* (present author's italics) in size than the original material, was confirmed in every test on Sands Nos. 2 to 8 by visual observation of the individual particles in motion. In every instance it was noticed that the first particles to begin moving were the larger ones, and that as the depth was increased, more of the smaller began to be moved." The grain sizes used in these experiments had an average diameter of 0.21—0.54 mm. In experiments with a grain size of 4.1 mm (Sand No. 9) there was found to be a perceptible though slight tendency for the *finer* fractions of the mixture to move first (cf. pp. 100, 101, and 115).

CHANG (1939, p. 1272) observed that "at the stage of initial dragging or rolling, . . . the coarser material was being dragged or rolled more rapidly than the finer . . . Hence, the coarser particle is more easily and quickly moved than the finer one . . . In experiments on transportation of a graded sand, it is usually found that a large percentage of the coarser constituents is sorted out and transported."

Similar observations have been made by a number of authors, e.g. GILBERT (1914, p. 178), HJULSTRÖM (1935, p. 301).

SCHAFFERNAK (1922, pp. 19—20) also points out that "in the case of a uni-granular material, especially one of large grain size, the individual grains move almost wholly by rolling over the bed and the movement is irregular because of the varied shapes of grains. In the case of well graded mixtures the coarser grains exhibit a decided sliding motion —actually are pushed—across the relatively smooth bed. This is also the reason why graded materials have such a high capacity even if relatively little fine material is contained in the mixture. The so-called effective grain size is therefore not in the range of the larger materials, but rather it is found among the smaller grain sizes in the mixture."

The above-cited observations on the initiation of movement and transport of particles in a graded material may appear to contradict one another to some extent. And the interpretation of them has in fact given rise to some difficulties.

An analysis of the entrainment process in a sediment mixture must begin with the two questions: 1) How is the flow in the layer nearest the bottom modified by the presence of different grain sizes, and what effect has such modification of the flow as may occur on the tractive force exerted by the water on the river bed? 2) How is the equilibrium of individual grains altered by the presence of grains of different sizes, and how does a possible change in their equilibrium affect their resistance to the flow?

As regards the first question it may be stated that the bed material affects the flow mainly by imparting to the surface a certain roughness. If the coarsest grains in a sediment mixture are almost entirely exposed they will determine the roughness of the bed, and the equivalent roughness k_s will then be the same as the diameter of the largest grains. The magnitude of the boundary shear or the tractive force will accordingly be greater than if the medium-sized grains had determined the roughness, assuming that the velocity at a certain depth is the same in both cases (cf. eq. 41).

If, on the other hand, the finer particles of the mixture fill the interstices between the larger ones, the roughness of the bottom is lowered, and likewise the tractive force. The latter effect is particularly pronounced when the grain sizes are such that filling of the interstices with fine material causes the flow to change from a rough or transitional type to smooth flow, accompanied by the formation of a laminar sublayer.

Addition of material with another grain size to a previously homogeneous sediment can thus either raise or lower the roughness of the stream bed, depending on the relative proportions of the different fractions and the arrangement of the grains. As already mentioned (p. 148), EINSTEIN found that in a not too heterogeneous mixture the grain size that determined the roughness of the bottom is "that sieve size of which 65 % per cent of the mixture (by weight) is finer".

The other question, that concerning the equilibrium of individual grains, can be discussed along similar lines. If the coarser grains are freely exposed they are without the support from adjoining particles which they have in a homogeneous material. They are therefore more easily moved by the current. This is especially true of coarse grains lying on a bed of considerably finer material. They are then set in motion and transported more easily than the fine grains, and also acquire a greater speed than the fine grains, due to the fact that they extend up into water layers with a greater velocity of flow (cf. SCHAFFERNAK 1922, pp. 19—20).

If the coarse grains are imbedded among the fine grains they are unaffected by the flow until the fine material is removed by the stream, leaving the coarse grains in a more exposed position. If the fine fractions of the mixture are so fine that cohesive forces are involved, the constraining forces of course increase still more.

The finest fractions of a mixture may be almost completely hidden between the coarser particles. Their position is then stable, and they are unaffected by the flowing water. In natural sediments there is always a certain proportion of such "hidden" particles. According to EINSTEIN (1950, p. 6) "these small particles are caught accidentally between the larger ones rather than primarily deposited by the flow itself. This is also suggested by the fact that the volume of the entire deposit does not change if the fine particles are eliminated from it; they merely occupy the voids between the larger grains."

In the light of these considerations the process of entrainment and transport of material in a mixed sediment may be described as follows, in analogy with the summary on p. 179.

1. For very coarse material (down to 6—8 mm grain size) the flow is completely rough, and the turbulence extends down into the spaces between the superficial particles. Even the smaller particles in the mixture are then subjected to the action of the flowing water. The roughness of the bottom and the turbulence is increased on account of the shape resistance of pebbles or boulders, and the eddies around the larger stones contribute to the entrainment and transport of the finer material.

Unless the flow velocity is very high the coarsest particles are moved only if the smaller particles that support them are removed by the stream. Their equilibrium is disturbed when the fine material disappears, and the stones roll or slide a short distance with the stream untill they lodge fast again. An "*erosion pavement*" of residual material is formed ("*shingling*"), and around the largest stones or boulders a characteristic morphological pattern of erosion and deposition formations often develops.

Thus in coarse mixed material (larger than 6—8 mm) the fine fractions are most easily entrained and transported.

2. For smaller grain sizes (the equivalent grain size less than 6—8 mm but larger than 0.3 mm) the main flow is still rough or transitional, but the turbulence does not fully extend into the interstices between the particles. Consequently the more shielded grains are not affected by the turbulent flow, while exposed grains are subjected to relatively greater force. The finer fractions are for the most part not entrained and transported

during the initial stages of the erosion process. The concentration of the flow resistance to the most exposed grains is most marked for grain sizes of 0.8—1.0 mm. At 0.3 mm grain size the grains are all within the laminar sublayer. In this latter case it is clear that the largest grains are most easily entrained during the initial stage of erosion. But this must also apply for grain sizes up to 0.8—1.0 mm.

For grains between 0.8—1.0 mm and 6—8 mm the turbulence extends partially down among the particles. In the initial stage of erosion the smallest grains reached by the turbulent flow should therefore be most easily dislodged. These grains are often not the smallest nor the coarsest in the mixture but the medium-sized. The tendency for a medium fraction to be selected may be expected to be particularly prominent for grains in the size interval 2—4 mm. For fully developed transport the coarser particles have a definite tendency to slide over a bed of finer particles. This is particularly true when the main flow is no longer fully rough, *i.e.* for grain sizes under 2 mm.

Thus in mixed sediments between 0.8—1.0 mm and 6—8 mm the fraction most easily entrained is one of medium grain size, commonly 2—4 mm. When transport of the sediment is fully developed the coarser material can slide over a bed of finer material. In mixed sediments between 0.3 mm and 0.8—1.0 mm the coarsest fraction is most easily entrained. It may then slide over a bed of finer material.

3. For grain sizes less than 0.3 mm the flow is smooth, and the process of entrainment depends largely on the content of cohesive material in the mixture. *The coarsest grains are most easily moved. After entrainment the material is transported for the most part in suspended form, with the exception of the very coarsest particles, which are transported as bed load.*

The laboratory results previously referred to are in complete agreement with the above account of the processes of entrainment and transport of a mixed sediment. We may draw the perhaps somewhat confusing conclusion that although, according to the erosion curve, material with an equivalent grain size of 0.2—0.5 mm is more easily eroded than any other equivalent grain size, it is not grains of size 0.2—0.5 mm which are most easily set in motion in unsorted material where other fractions are also present. Instead it is grains of size 1—6 mm that are least stable in a mixture. It is these latter grains that are most easily set in motion when bed load begins to be transported, and they are the last to come to rest when the movement ceases. One can say that they constitute a fraction that is seldom allowed to rest by flowing water. It will be seen in a following section (p. 191) that this circumstance is of great importance for the interpretation of geological and stratigraphical observations.

Particle shape

In the preceding discussion it has been assumed that the grains have a well-rounded shape. However, there are certain mineral particles or rock fragments (e.g. mica flakes, shale fragments, or crushed rock particles) which are often either sharp-edged or flat, and it may therefore be appropriate with a brief discussion of the influence of particle shape on the critical velocity for erosion.

It is obvious that the shapes of individual particles affect both the flow near the bottom and the particles' resistance to motion. It has been stated by several authors (e.g. CASEY 1935, p. 7, and SHIELDS 1936, p. 14) that sharp-edged grains offer a greater resistance to flow on account of their greater shape resistance than do well-rounded grains of the same size. Flowing water therefore exerts a greater force on them.

However, experiments by SCHOKLITSCH (1914, quoted in CASEY 1935) show that thin flakes of well-laminated shale require a slightly larger tractive force for entrainment than does ordinary natural bed material. The same tendency is exhibited by sharp-edged grains of lignite, according to CASEY, and by crushed granite gravel and sharp-edged amber grains according to SHIELDS.

It is therefore probable that sharp-edged or flat grains offer a somewhat greater resistance to motion on account of a greater friction between grains and less pore space, and that this greater resistance outweighs the relative increase in the tractive force acting on them.

It is also worth noting that sharp-edged or flat particles are more easily borne along in suspended form than are rounded grains, because their settling velocity in water is less (cf. KRUMBEIN 1942, NEVIN 1946, and ALBERTSON 1952). One may often observe how relatively large mica flakes are swept along by flowing water, while considerably smaller quartz grains remain on the bottom unmoved.

In conclusion it may be stressed that *the influence of particle shape on the critical erosion velocity is insignificant in comparison with the influence of grain size and density. Usually a slight increase of the erosion velocity may be expected for sharp-edged and flat particles.*

Erosion by silt-laden water

If a stream is carrying a considerable quantity of suspended material, some of this will fall out of suspension and fill the spaces between grains of sand and stones on the bottom. The effect of such deposition is two-fold. The roughness of the bottom diminishes, and the coarser particles become more firmly fixed in position, particularly if the deposited material is mainly fine. Material of the category "fine silt" imparts a certain cohesion to the stream bed, and naturally raises the resistance to erosion. Both factors act in the same direction, and the result is an increase in the critical erosion velocity.

Besides the two factors already mentioned there is a third which may be important when the concentration of sediment is very high. Under such conditions, particularly if there is much coarse and medium silt in suspension, density stratification may occur (cf. p. 156), the turbulent exchange and the turbulent velocity fluctuations are damped, and the erosive and transporting capacity of the stream thereby lowered. A turbidity current of non-colloidal silt is in consequence generally depositing rather than eroding.

The erosive capacity of a sediment-laden stream has been investigated by, among others, FORTIER and SCOBEY (1926), LANE (1935), and WRIGHT (1936). FORTIER and SCOBEY (referred to by HJULSTRÖM 1935 and 1939, MAVIS, HO, and TU 1935, and BROWN 1950) determined the permissible non-scouring canal velocities for various types of bed material, taking into account among other things the presence of suspended sediment and the age of the canals. They found that "the silts will precipitate under the reduced

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velocities and possess sufficient colloidal matter to form a plastic, highly cohesive mass". The colloids "will make the bed all the more tough and tenaceous, increasing its resistance to erosion".

If the original canal bed consisted of gravel, sand, or silt, the permissible velocity increased by between 30 and 100 % for water with a high sediment content. If the bed was of cobbles or shingles the effect was rather less, probably because deposition of fine material between the stones was then counteracted by the flushing action of the current. If the canal bed was highly resistant to erosion to begin with (e.g. if it was made up of shales or hard pans) the suspended material had no effect on erosion.

Finally, FORTIER and SCOBEY observed that if the water bore particles that might have an abrasive effect on the bottom (sand, gravel, or rock fragments), then the resistance to erosion of, for instance, a bed of shales or smooth, compact clays decreases, thereby lowering the permissible canal velocity.

In brief, it may be said that *the presence of suspended material in flowing water raises the critical erosion velocity for a bed of coarser material, but that coarse suspended material or bed load may lower the critical velocity for a bed of fine material that was previously resistant to erosion.*

Application to natural streams

The curves for the erosion velocity and the subsequent discussion of various modifying factors have mainly been based on laboratory observations, and consequently it cannot be assumed that the conclusions arrived at can be directly applied to natural streams. There is a considerable difference between a geometrically well-defined flume with an almost constant flow of water and a natural watercourse with its irregular hydraulic geometry and often highly variable flow conditions. In a natural stream there is a continual alternation of scour and deposition of bed material and suspended sediment.

However, there is no doubt that the resistance of particles to erosion and the process of entrainment are the same in both cases. Deviations from the derived rules relating flow velocity and erosion of various kinds of material are principally due to the different configuration of flow in natural streams compared with the flow in a flume.

One of the assumptions made in deriving the erosion curves was that the velocity fluctuations at a point near the bottom are directly proportional to the mean flow velocity at the same point. If the flow velocity increases, the velocity fluctuations should increase in the same proportion. This does in fact appear to be true of the regular parts of a river, according to KALINSKE (cf. p. 151).

But the real situation is often considerably different. In extreme cases it may even happen that scouring occurs at a place where the mean velocity is zero. An example of this is where a jet of water is directed perpendicularly on to a river bed. If the jet oscillates slightly the mean velocity may be zero, but nevertheless the bed may be severely eroded.

It is thus obvious that the local state of turbulence determines where erosion and transport occur. Macroturbulence is particularly important. Where there are obstacles of some kind (bridges, jetties, poles, boulders, small promontories) the turbulence and velocity fluctuations are greater, and there may be local scouring, in spite of the fact that the mean velocity is not so great as normally to give rise to erosion. It is therefore important to study macroturbulence when investigating erosion in an actual case (cf. p. 161 and p. 256).

If we disregard places where the local turbulence is extremely strong, and confine the application of the curves to reaches of natural watercourses where the topography of the bed and banks is regular and even, their validity appears to be good, in the experience of the author. For instance, it is a well known fact that "deep canals can safely withstand higher velocities than shallow ones" (BROWN 1950, p. 809), a fact that is at once evident from the curves and corresponding equations. The few values of erosion velocities in natural watercourses of various depths that are stated in the literature (see, e.g. SUCHIER 1883, SCHOKLITSCH 1914, RUBEY 1938, ARNBORG 1948), seem to confirm the applicability of the erosion curves even to natural watercourses.

SOME GEOLOGICAL AND MORPHOLOGICAL APPLICATIONS

Discontinuities in the grain size distribution of fluvial deposits

It has often been pointed out that certain grain sizes are less common than others in fluviatile sediments (PETTIJOHN 1949, p. 41). The interval of grain sizes 2—4 mm is notably less frequent compared with both coarser and finer material. This comparative scarcity reveals itself in two ways: there is a definite deficiency of material in this grain size interval, as may be seen when the analyses of a large number of representative samples are considered as a whole, and it is seldom that an individual sample has its maximum frequency in the interval 2—4 mm. Samples containing material both finer and coarser than 2—4 mm in fact often have a frequency minimum between 2 and 4 mm, with a primary and secondary maximum at coarser and finer sizes. Samples with both a primary and a secondary maximum are commonly referred to as *bimodal*. Two analyses from the author's investigations in the valley of Klarälven may be taken as typical bimodal grain size distributions (fig. 15).

The reasons for the lack of material in the interval of grain sizes 2—4 mm and for the

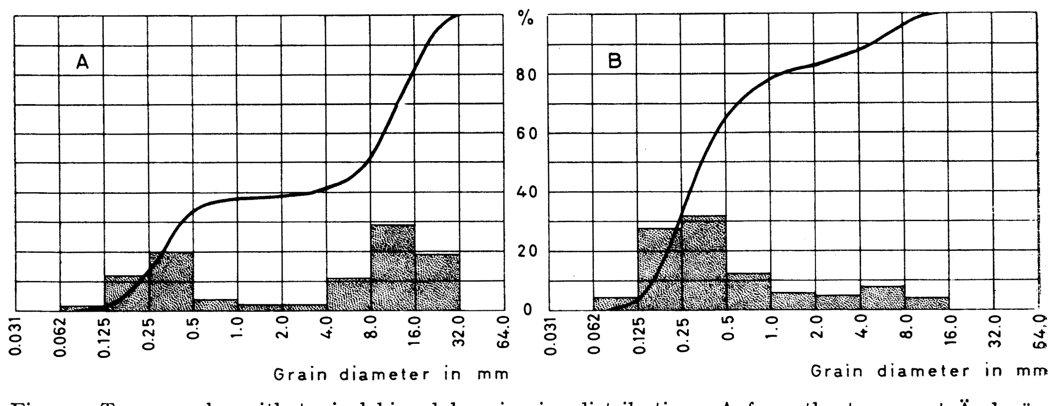

Fig. 15. Two samples with typical bimodal grain size distributions. A from the terrace at Ändenäs in Norra Ny parish, and B from the recent river bed at Bergsäng in Ekshärad parish.

occurrence of bimodal distributions have long been discussed in the geological literature, for instance, by UDDEN (1914), WENTWORTH (1933), EINSTEIN, ANDERSON, and JOHNSON (1940), and HOUGH (1942). PETTIJOHN (1949) has recapitulated the various attempts at explanation, and concludes that a definite solution of these problems is still far off.

A review of both previously published and newly collected information on samples with bimodal grain size distributions has been carried out by L. ARNBORG at the Geographical Institute in Uppsala, and the results will be published soon in Geografiska Annaler. Here the problem will be discussed only in connection with the presentation and analysis of the erosion curves.

Three main possible explanations of bimodal distributions have been put forward. 1) The sample was taken in such a manner that material from two different strata with different grain size distributions was included. The overall distribution may then acquire two maxima, corresponding to the maxima of the two components. 2) The process of weathering of the source rock and the process of wear during transport produce a sediment where certain grain sizes are more common than others, mainly because some grain sizes, among them the 2—4 mm, are mechanically unstable. 3) The selective transportation of material by flowing water for some unknown reason gives rise to bimodal frequency distributions with a minimum in the interval 2—4 mm.

The first explanation may of course often be true. But it cannot explain why the frequency minimum so often lies in the range 2—4 mm, nor why there seems to be a definite deficiency of material in this range.

The second explanation could account for the deficiency of 2—4 mm particles. But it is difficult to see why the selective transport process could not compensate for a natural deficiency in a particular interval of grain size, and relatively often produce fluvial sediment with a maximum in the interval concerned. Finally, it appears that a bimodal frequency distribution in a fluvial sediment may occur equally well if the source rock is coarse-crystalline intrusive rock as if it is fine-crystalline lava rock, or some sort of sedimentary rock (ARNBORG). It would be expected that mechanical stability should be dependent on the size and character of the individual mineral grains in the source rock. PETTIJOHN (1949, p. 45) points out that "another factor to be considered is the nature of the material produced by the weathering. Some rocks yield *blocks* upon breakdown, whereas others undergo granular disintegration and yield *grains* of sand size." It may then be stressed that the mechanical instability of grains in the grain sizes 2—4 mm is not likely to be considered as a general explanation for the discontinuities in the grain size distribution.

The third explanation seems plausible, since flowing water is certainly an effective sorting agent. But the details of the explanation have proved difficult, and no unobjectionable theory has been put forward. The assumed general deficiency of fine gravel has been difficult to explain, and moreover a satisfactory interpretation of the circumstance that the frequency minimum is in just the 2—4 mm range has been hard to obtain.

Some further views on the problem will be put forward here, based on the discussion on pp. 185—188.

To begin with it may be remarked that while erosion is in progress along a certain stretch of a river, an "erosion pavement" or shingled surface is formed there. Samples taken from such a region may of course exhibit a bimodal distribution of grain sizes with one maximum in the interval for the residual material and another for the original material. A bimodal distribution may also be obtained where coarser material has slid or rolled over a bed of finer material. Although these two types of bimodal distributions are not uncommon, they are undoubtedly not the only types, and the primary question still remains to be answered—namely, why the intermediate grain sizes are absent.

On p. 188 it was stated that in a mixed sediment with grain sizes between 0.8—1.0 mm and 6—8 mm it is an intermediate fraction that is most easily entrained, commonly the range 2—4 mm. The least stable particles in a mixture (in the sense: least resistant to movement by flowing water) are those of grain size 1—6 mm. It is these grains that are most easily dislodged when transport of bed load commences, and they are the last to come to rest when the movement ceases.

If this conclusion is correct, it is obvious that flowing water tends to sort out and carry forward material of the class fine gravel. Wherever mixed material containing sand and gravel is exposed to the action of flowing water it is the fine gravel that is most easily removed from the mixture. Material subjected to such action—fluvial sediment or perhaps moraine—may thus readily acquire a bimodal grain-size distribution owing to selective erosion and transport.

This at once raises the question: what happens to the selected material? It must be deposited somewhere, and one would expect the deposit to be monomodal, with a frequency maximum in the fine gravel range.

It was mentioned above that the particles most easily set in motion are those in the size interval 1—6 mm. Whether they are mixed with finer material or with coarser they are sorted out and carried along by the stream. They are, so to speak, hunted out from the bed of a stream. Let us consider the situation where fine gravel has been deposited on a part of a river bed. Under what conditions may other material be deposited on top of this layer, thus preserving it? If coarser material is transported from upstream as bed load, the flow velocity will as a rule have to be so high that the previously deposited fine gravel is set in motion. The same often applies if finer material is brought down as bed load. The only sure possibility of further deposition and consequent preservation of the fine gravel layer is the deposition of suspended material. This possibility is certainly rather seldom realised.

The conclusion to be drawn from this reasoning may be put thus: when gravel grains have been worn down to a size of about 5—6 mm, the transportation of them by the stream becomes more relentless, and they are often prevented from coming to permanent rest until they have been worn down to a size of 1—2 mm or less. This may well be an important cause of the general deficiency of particles in the interval 1—6 mm.

For comparison it may be mentioned that there appears to be a similar deficiency of fine gravel in beach sediments (PETTIJOHN 1949, p. 42), which is explicable in view of the fact that the transport process is much the same on a coast as on the bed of a river. In eolian sediments, on the other hand, the grain sizes 0.06—0.12 mm have been found to be less frequent (PETTIJOHN 1949, p. 44). A reference to the erosion curves in fig. 177 will show that it is just these sizes that are most easily eroded. In this connection it should be pointed out that the formation of a laminar sublayer does not have the same significance in eolian processes as in fluvial, since the ground surface is rough to a wind even down to very small grain sizes. It is likely that in selective eolian erosion the boundary between bed load or the saltation phase of movement and the suspension load is of greater importance, since this boundary is close to the lowest possible erosion velocity, and since the suspension load (eolian silt or loess) is transported a much greater distance than fluvial sediments.

To summarise, it may be said that *grains of the class "fine gravel" are "hydrodynamically unstable", and this may give rise to bimodal frequency distributions and to a general deficiency of material in this size class. Imperfections in the method of sampling and mechanical instability may both lead to the same consequences, however, and it is probable that all these factors contribute to the observed situation.*

The relation between grain size and the longitudinal profile of a river

Study of the relation between the bed material and the longitudinal profile of a river has led to the view that the gradient of a river in equilibrium is directly proportional to the grain size in stretches where the river does not undergo marked change, for instance, owing to confluence of a tributary (cf., for instance, SHULITS 1941, p. 622).[1] Since it has been found empirically that the particle size on the whole follows STERNBERG's abrasion law (STERNBERG 1875)—which implies that the grain size decreases exponentially down the stream—a river in equilibrium should therefore have an exponential longitudinal profile. In actual fact this has been found to be generally more or less true.

However, some recently published papers have reported interesting deviations from the exponential profile. As there is an intimate connection between these deviations and the discussion in the preceding section, it seems appropriate to devote some attention to these investigations.

YATSU (1955) has studied the longitudinal profiles of some "main rivers in central Japan, whose lower courses have neither tributaries nor distributaries and so may be regarded as graded rivers". Seven of the nine rivers investigated were found to have "slopes consisting of two exponential curves".

Examination of the median diameter of the fluvial deposits in relation to the distance revealed that the bed material in the above seven rivers exhibited a discontinuous transition from gravel to sand. As the grain size decreased downstream sizes 2—10 mm were found to be relatively infrequent. In the transition region between coarser and finer material samples of the bed material were in general found to be bimodal. Of the two rivers

[1] However, this view has received justifiable criticism. It has been pointed out that many other factors are also operative (cf. MACKIN 1948, p. 482).

that had longitudinal profiles corresponding to single exponential curves, one had no median grain sizes less than 10—20 mm, and the other no median grain sizes coarser than 2 mm.

YATSU (p. 661) has drawn from his investigations the conclusion that "if a river attains the graded state by degradation or aggradation, its longitudinal profile has a portion of discontinuous decrease of stream gradient and will be divided into two exponentials, because of the discontinuous collapse at the grain size from two to four mm which seems to occur in streams. Sternberg's law is correct on the river course, deposits of which are only gravelly or only sandy and silty."

Similar conclusions have been drawn by YATSU (1954, p. 63) from studies of the slopes of alluvial fans in Japan: "And the slope discontinuity found out at the base of an alluvial fan is due to the discontinuity of size frequency distribution of the debris transported by the rivers flowing on it. The deposits of an alluvial fan consist of gravel and sand size, but their mean diameter is of gravel size, while the deposits of a flood plain or delta range from sand to clay. The deposits of granule size are of small quantity everywhere, so the slope in correspondence to such particle size is wanting between the base of an alluvial fan and the adjacent flood plain or delta."

Regardless of whether the deficiency in material in the size range 2—4 mm is attributable to mechanical or to hydrodynamical instability of particles in this range, it is undoubtedly of great interest that this deficiency can have a direct morphological consequence, as YATSU has shown.

The sorting and segregation of heavy minerals in water-borne sediments

The study of accessory heavy minerals in fluvial, eolian, or littoral sediments has often proved to be a valuable method for determining the source of a particular sediment. The method is based on the fact that certain minerals are typical of definite parent rocks.

It has been observed that the frequency of a particular heavy mineral in a sample of sediment often cannot be directly correlated with the frequency of the same mineral in the source rock, however. The frequency appears to be highly dependent on the grain size distribution of the sediment sample. The heavy mineral is generally concentrated in the finer fractions of the sediment.

In some cases a high degree of mechanical enrichment of heavy minerals takes place. The "*black sand*" that occurs on the coast, in river beds, and in sand dunes is a local concentration of material bearing relatively stable ore, such as magnetite, haematite, or ilmenite. Sometimes the deposits are sufficiently large to be workable.

It is clear that the mechanical sorting and redistribution of various mineral components in a sediment by selective erosion, transport, and deposition may lead to such radical changes that the original distribution of minerals is completely concealed. The value of an examination of heavy minerals as a diagnostic method therefore depends on how far it is possible to judge the effect of the mechanical sorting in different environments.

RUBEY (1933) has an interesting discussion of the factors that determine the distribution of heavy minerals in a sandstone. He comes to the conclusion (p. 3) that "factors such as differences in the density and hardness of the various minerals, differences in the original size of the various mineral grains in the source rock, the amount of abrasion that all grains have undergone during transportation, the different settling velocities of the various grains at the site of deposition, and the degree of sorting to which all grains were subjected there—all these factors seem competent to cause large variations in the relative abundance of various minerals in different samples of deposits that have been derived from exactly the same source rock and also in different size fractions of each of the samples. Certain of the factors, such as abrasion, tend to concentrate the heavier minerals in those samples that are predominantly finer grained; whereas other factors, such as sorting, tend to concentrate the heavier minerals in the finer grained portions of each sample, regardless of whether that sample is predominantly fine-grained or coarse-grained."

RUBEY's analysis is based on the effect of density on the settling velocity of the particles in water. Since there are also other factors than the settling velocity in water that determine the transport of sediment, we will here consider some factors affecting the sorting process for a non-suspended material, on the basis of the previous account of the processes of entrainment and transport of bed load. Since this treatment is not intended as an exhaustive analysis of the processes that determine the distribution of heavy minerals in a sediment, no attempt is made to discuss such factors as the size of the original grains of mineral in the parent rock, or differences in the resistance of particles to chemical and mechanical weathering or to abrasion during transport.

During the deduction of the formulas for the critical erosion velocity (see, for instance, p. 175), it was observed that the greater the density of a material, the greater must be the velocity of flow to set in motion and transport particles of the material in question. In accordance with formulas (48) and (49) it is possible to construct erosion curves for material of any density.

Fig. 16 shows curves for the critical erosion velocity for four different minerals: magnetite with density 5.2 g/cm^3, amphibole with density 3.3 g/cm^3, quartz sand with density 2.65 g/cm^3, and lignite with density 1.25 g/cm^3. The curves all assume that the depth is 1 m (or, rather, that the flow velocity is measured one metre from the bottom), but it is of course easy to obtain curves for other depths in accordance with fig. 13. Other factors involved have been determined in the same way as for fig. 13. Corresponding curves for eolian erosion are included for purposes of comparison.

It will be seen from the curves that the critical erosion velocity varies markedly with the density of the material. For instance, if we compare different materials of grain diameter 4 mm, we find that homogeneous quartz grains are eroded when the flow velocity is 110 cm/sec. 1 m above the bottom, while amphibole requires 140 cm/sec., and magnetite 190 cm/sec. Lignite, on the other hand, requires only 40 cm/sec. It may be seen that the relative velocity differences are much less in air than in water, which is to be expected in view of the buoyancy force in water. This circumstance helps to make the sorting process less effective in air than in water.

We may carry out an analysis of the curves analogous to that carried out in the treat-

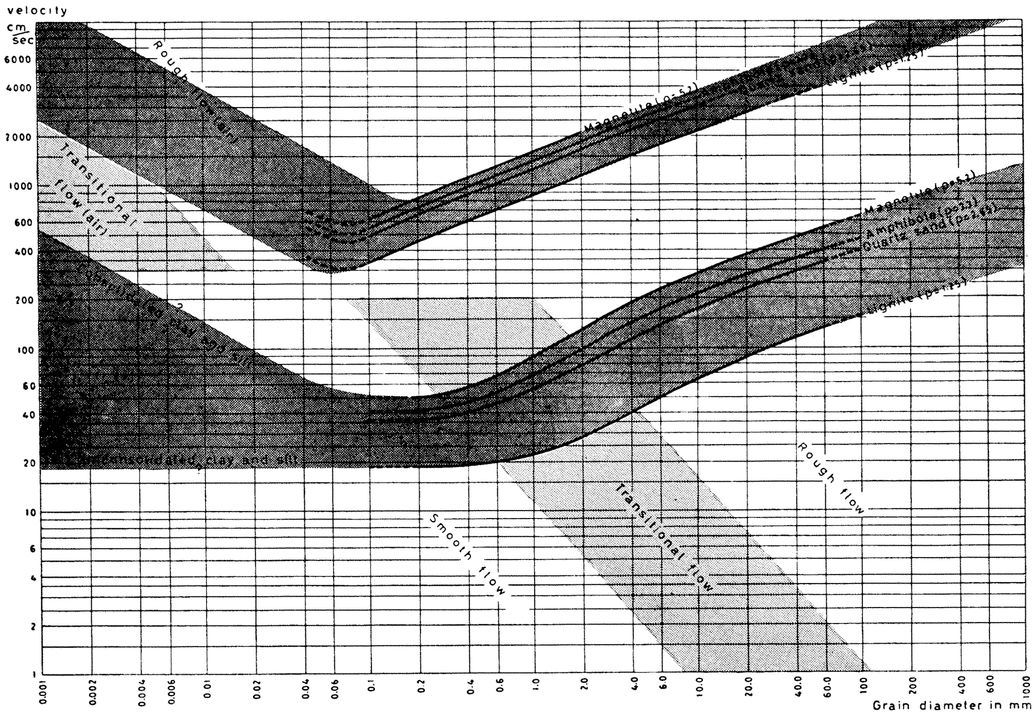

Fig. 16. Curves showing the relation between particle size and critical erosion velocity for uniform materials of different densities. The flow velocity is that 1.0 m above the river bed. Curves for the critical erosion velocity for eolian erosion are included in the upper part of the diagram.

ment of erosion of a sediment mixture and in discussing the origin of bimodal grain-size distributions. Let us assume that the material concerned contains both quartz and magnetite particles in various grain sizes.

1. For very coarse bed material (of equivalent grain size at least 6—8 mm) the flow is rough, and all the superficial grains are subject to the action of the water, even smaller particles in the interstices between the larger. If all the particles had the same density an erosion pavement consisting of the coarsest grains would be formed. When there are different densities smaller particles of higher density have the same resistance to erosion as larger particles of lower density. For instance, according to the erosion curves a quartz particle 20 mm in diameter corresponds to a magnetite particle about 6 mm in diameter. The erosion pavement will therefore consist of coarser and less dense quartz particles together with finer and denser particles, roughly in the size ratio mentioned. The effect may be somewhat enhanced in that the coarser quartz particles project up into flow layers of greater velocity. There is consequently a size-sorting of the material on the stream bed implying an arrangement such that all the particles in a particular part of the stream bed offer roughly the same resistance to erosion.

2. When the grain size is smaller (equivalent grain sizes less than 6—8 mm), the turbulence

does not extend right down between the individual grains. So the smallest grains, which naturally include the heavy grains of magnetite, are therefore partially protected from the action of the turbulent water. Whereas in the coarser fractions they were carried forward or deposited together with coarser grains of lower density, they are now more often left behind in the movement downstream. They remain stagnant on the river bottom, and there is some tendency to enrichment as more material is brought down from upstream.

This tendency is the more marked as the grain size diminishes. When the equivalent grain size is less than 2—4 mm both the higher density and the smaller grain size contribute to enrichment of the denser fraction. Only when the flow is so powerful that the entire bottom layer moves forward are the magnetite grains transported farther.

It will be seen from the diagram that when the flow velocity is between 35 and 50 cm/sec. no magnetite grains are set in motion, while quartz particles of sizes up to about 1 mm can be carried forward by the current. If the flow velocity is between these limits the erosion and transport of quartz grains is accompanied by an enrichment of the heavy mineral. An erosion pavement of heavy residual minerals is formed. However, in a natural stream the variability of the flow tends to counteract this enrichment, while at the same time the continual process of wearing leads to some dispersal of small fragments in suspension.

As will be seen in a following section, the boundary between bed load and suspended material also depends on the density of the grains concerned. For grain sizes near this limit grains of quartz may be taken up in suspension, while equally large grains of magnetite remain on the bottom. This also contributes to enrichment of heavy minerals in grain sizes near the limit.

A comparison between other minerals can of course be carried out in a similar way to that for quartz and magnetite grains.

Other factors often lead to deviations from the processes outlined above. One circumstance is that the density of accessory mineral particles seldom affect the density of rock fragments which are composed of a large number of mineral particles. Not until grain sizes are reduced to such an extent that a large proportion of the grains on a river bed are individual mineral particles does the process of sorting and enrichment become effective.

It should be stressed that *it is as yet impossible to estimate the relative proportions of minerals of different densities in bed load on the basis of the composition of the original rock.*

It is possible to make an estimate of the relative concentrations of various heavy minerals in a deposit of *suspended material*, however, assuming that the flow situation and the relative concentrations of the minerals in at least one site are known (p. 221).

The conclusions that may be drawn from the above reasoning may be expressed as follows. *In material transported as bed load, as in suspended material (cf.* RUBEY), *the grain size distribution for denser material is displaced towards finer sizes. The displacement can be computed approximately from the diagram in fig. 16. For material under 2 mm a more pronounced enrichment is possible. The upper limit of grain size for the enriched material is generally determined by the normal size of the individual mineral grains in the parent rock, and the lower limit by the size at which material may go into suspension. In the enriched material the denser particles are smaller than the lighter ones in the mixture.*

There is in the literature on sedimentology a large amount of information about the frequency of heavy minerals of various grain sizes in natural sediments. One of the most thorough investigations was carried out by RITTENHOUSE (1943), who analysed sands from the Rio Grande in New Mexico. Other informative papers are those of MACKIE (1923) from investigations in north-eastern Scotland, and of RUSSELL (1936) from the Mississippi.

Two analyses of samples of black sands with a very high percentage of heavy minerals taken from the author's own investigations may serve to illustrate the preceding discussion.

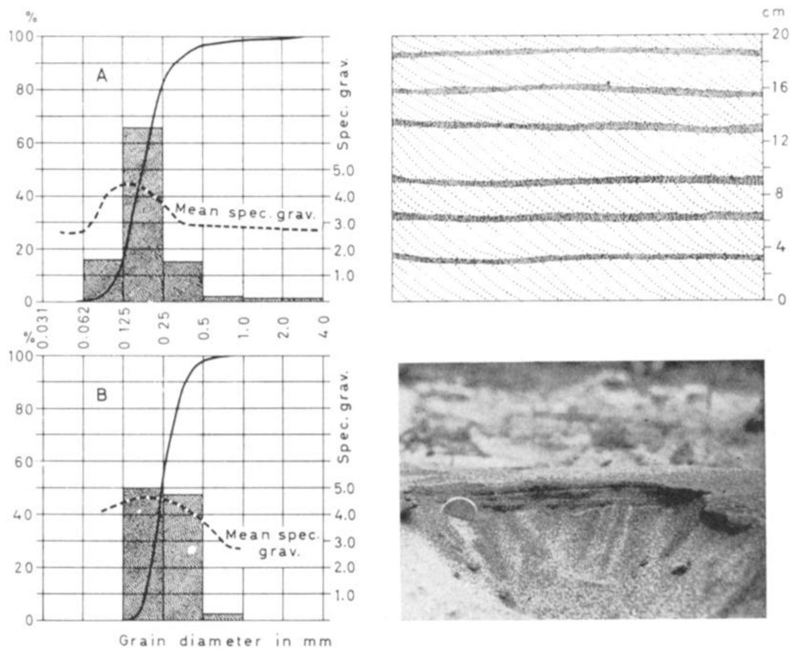

Fig. 17. Analyses of two samples of black sand with a high percentage of magnetite. A is an analysis of a river sediment from Baskenäs in Norra Ny parish. The stratigraphical situation is shown in the sketch to the right. B is an analysis of a beach sediment from the lake Mangen south-west of Filipstad. The photograph to the right shows the alternation of lighter and darker laminae of different magnetite concentration. Note the piece of money to the left. In both diagrams the average density of the sediment for each fraction is marked (Photo G. Zetterberg 1956).

Fig. 17 A is for a river sediment from Baskenäs in the valley of Klarälven (see Pl. 1). The sediment is mainly composed of sand transported as bed load. The parent rock for these deposits is mainly granites and gneisses of Archaean age, with an original ore content of at most 1—2 %, but normally some tenths of a per cent (cf. p. 240).

The stratigraphical situation is shown in the diagram on the right. The type of stratification may be described as a nearly tabular arrangement of cross-laminated units. The thickness of the individual strata varies between 2 and 5 cm. The laminae in each stratum have a dip of about 30°. The greater part of each stratum consists of quartz grains mixed with felspar and slight amounts of accessory heavy minerals. Between these light-coloured strata there are thin laminae (0—5 mm thick) of dark mineral grains which are mainly magnetite.

The sample analysed originated in one of these dark laminae. The analysis shows that the median grain size is 0.18 mm and the sorting coefficient 1.27.[1] No exact determination of the mineral content in different size fractions has been carried out, but instead the average density has been determined for each fraction, since this is a good and easily-determined measure of the degree of enrichment.

As expected, the heavy mineral is concentrated in the finer fractions. The maximum density is 4.4 g/cm^3, which implies an ore content of 80 % or an iron content of 55—60 %. The coarsest fractions consist almost entirely of light minerals or rock fragments. A noteworthy fact is that the finest fraction, under 0.06 mm, contains very few grains of magnetite. However, this fraction is very small.

The formation of these laminae of enriched material may be interpreted in the following way. The layer sequence developed during a downstream movement of current ripples or small bars. The original deposit was truncated by erosion and transport on the proximal side of the ripple during the downstream movement. But the basal parts of the ripples were not moved, which means that the process occurred during a phase when there was a net deposition of material in the region. The material arriving from upstream must have contained a good deal of heavy mineral, presumably enriched after several phases of redeposition.

A layer of residual material consisting mainly of particles of magnetite, was formed on the proximal side of each ripple. Some of the successively enriched material was carried up over the crests of the ripples, whereupon it was deposited in the bottoms of the troughs. According to this interpretation, the dark layer of heavy minerals is a combination of material enriched in the truncated erosion surface and of grains deposited in the troughs. The flow velocity must have been near the limiting value for erosion of magnetite grains (cf. fig. 16) during a considerable period.

The high percentage of light minerals among the very smallest grains may be due to deposition of suspended material from the current. Traces of such deposition are found as a subsidiary fraction, *wash load*, in most sand samples, even though the sand may have been mainly transported as bed load.

Fig. 17 B shows sediment from the shore of the lake Mangen, about 20 km south-west of Filipstad. The locality is situated in the distal region of Brattforsheden, the large sub-aquatic glacial delta described by HÖRNER (1927).

The black sand occurs as lenses which form a zone some metre wide on the gentle beach slope. The vertical thickness is at most about 20 cm. It is evident that these lenses were accumulated by the waves. A vertical section reveals an alternation of lighter laminae with more quartz and darker laminae with a very high content of iron mineral (see the photograph to the right).

[1] According to TRASK's definition the square root of the ratio of the quartiles, $\sqrt{Q_3/Q_1}$, where $Q_3 > Q_1$ (see, for instance, KRUMBEIN and PETTIJOHN 1938, p. 230).

The analysis gives about the same result as for the fluvial sample. The material is somewhat coarser (median size 0.25 mm), however, and somewhat better sorted (the sorting coefficient = 1.18). Particles coarser than 1 mm or finer than 0.06 mm are almost entirely absent. The content of ore attains a maximum of about 80—85 % by weight in the size interval 0.15—0.20 mm. The slight tendency to a decrease in the content of heavy minerals among the smallest particles may have the same cause as in the fluvial sediment.

The distribution of heavy minerals in this case is also what might be expected from the previous discussion. The high degree of enrichment must have occurred under the action of the waves in their motion to and fro. As the water moved up over the beach the flow was so powerful that all kinds of grains moved with the water. As the waves receded, on the other hand, the flow was weaker and only more easily movable grains went back with the water. The material was thus subjected to washing, which led to enrichment of magnetite and ilmenite.

The velocity limits for the action of the waves may be estimated from the curves in fig. 16, although a correction must be made for the depth of water. It should also be borne in mind that conditions of turbulence and the vertical distribution of velocity are not quite the same as in the fluvial case. But in principle the process of enrichment should be the same in rivers and on beaches. The alternation of dark and light laminae can be ascribed to variations in the effect of the waves.

The geometry of the stream channel

In the geomorphological analysis of fluvial land forms it is assumed that a river or river system gradually evolves to a state of equilibrium. When this state is achieved the river is said to be *graded*. The term has been defined in several ways. Perhaps the most adequate definition is that of MACKIN (1948, p. 471). "A graded stream is one in which, over a period of years, slope is delicately adjusted to provide, with available discharge and with prevailing channel characteristics, just the velocity required for the transportation of the load supplied from the drainage basin. The graded stream is a system in equilibrium; its diagnostic characteristic is that any change in any of the controlling factors will cause a displacement of the equilibrium in a direction that will tend to absorb the effect of the change."

The flowing water in a river is continually shifting the material of the bed. Sometimes scouring leads to a local deepening of the river channel, and sometimes the channel is made more shallow by filling. This process takes place in all watercourses, and is not to be confused with the slow and more continuous process that occurs in *degrading* or *aggrading* rivers when for some external reason the river changes its equilibrium position (cf. MACKIN 1948, p. 478). It is necessary to know the laws governing erosion, transport and accumulation of sediment, and the relations between the state of flow, transport of material, and the geometry of the stream channel, in order to be able to characterise a graded river. It thus becomes necessary to pay attention not only to the longitudinal profile but also to the equilibrium conditions for the transverse section of the river bed under various conditions.

MACKIN (1948, p. 484) has recognised this, and suggests an alternative extended definition of a graded stream. "A graded stream is one in which, over a period of years, slope *and channel characteristics* are delicately adjusted", etc. LEOPOLD and MADDOCK (1953, p. 46) favour this later version, on the basis of their investigations.

The study of the geometry of a stream channel may imply a descriptive examination of the morphology of the river bed as related to bed material and hydrology, or it may imply an analysis of river hydraulics as in the work of LEOPOLD and MADDOCK, or it may be an investigation of the hydrodynamic conditions for equilibrium in various situations. From the geomorphological point of view the common goal must in any case be to derive, from a knowledge of the regime of water discharge, sediment load, characteristics of bed material, and other relevant factors, the river pattern under graded conditions and to give a quantitative estimate of what effect the river may produce in a given situation. Conversely, a study of fluvial land forms should enable conclusions to be drawn regarding the process of their formation and the environment of their formation. We obviously have a long way to go before this goal is achieved.

In the opinion of the author an analysis on a hydrodynamic basis of the transverse section of a river under various conditions should be a gateway to further studies. The morphological action of a river is often dependent on a lateral displacement of the channel, and investigation of the conditions for lateral erosion, and of the relation between lateral and vertical erosion is therefore a task of primary importance. This entails an investigation of the conditions under which bank lines are stable, for different flow conditions and different materials.

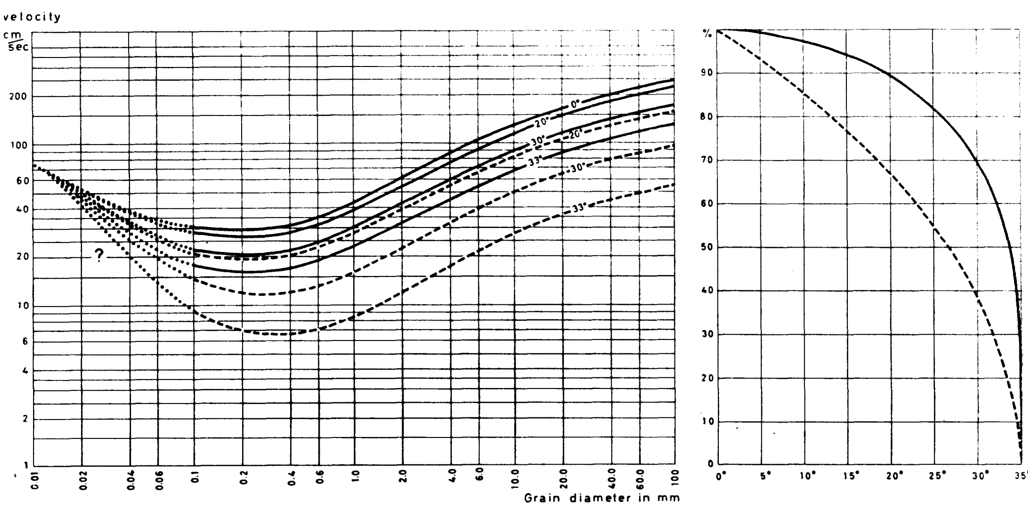

Fig. 18. Curves showing the relation between particle size and critical erosion velocity for uniform materials on different slopes (0°, 20°, 30°, and 33°). The flow velocity is that 0.1 m above the river bed. Continuous lines: flow parallel with the sloping surface. Dashed lines: flow straight down the slope. The diagram on the right shows the erosion velocity for frictional material as a function of the angle of slope and expressed as a percentage of the erosion velocity for a horizontal surface.

It is often pointed out that "a higher velocity is required to set in motion a particle on the bed than one on the sloping banks" (MACKIN 1948, p. 485). The main reason is that the position of individual particles on a sloping surface is not as stable as on a horizontal one.

The effect of slope on the critical erosion velocity may be seen from eqs. (46) and (47). Curves for the critical velocity on different slopes are given in fig. 18. The velocity is taken to be measured 0.1 m above the river bed. It has been computed both for the case where the flow is strictly horizontal and parallel with the sloping surface (continuous lines), and for the case where the flow is straight down the slope (dashed lines).

Since it has not been possible to determine any definite relation between the erosion velocity and grain size for cohesive material, the shape of the curves is uncertain for fine sediment (dotted lines). However, there can be no doubt that *the slope of the bottom is of no importance as regards erosion of purely cohesive material.*

In the diagram on the right the erosion velocity is plotted against the angle of slope. The erosion velocity is expressed as a percentage of the erosion velocity for a horizontal bottom. The diagram is of course intended only for frictional material.

As yet no laboratory experiments have been carried out to check the calculated values, and accordingly the quantitative analysis will not be carried further here. However, we may draw some qualitative conclusions.

It will be seen from the diagram that the critical velocity for erosion of sand and coarser material is considerably smaller if the surface slopes. The velocity for lateral erosion along a sloping bank is probably somewhere between the two curves in the right-hand diagram, since the direction of flow varies on account of turbulence. The transverse section of a river in equilibrium flowing in a channel of uniform material without transport of bed load ought to be such that the effect of the flow is somewhat below the critical value for both bottom and sides. The requisite profile is then flatter than the semicircular form that is most favourable for the passage of the water.

Transport of sediment occurs even in a graded stream, however. If bed load is to be transported the flow velocity must be higher than the critical value for erosion. This means that the transverse section must be still flatter. The flattening or broadening of the section implies that lateral erosion proceeds until the distribution of velocities over the transverse section is such that a state of equilibrium is achieved.

Enrichment of the coarser fractions in the bed of a river gives the bottom a higher resistance to erosion .This is particularly evident where a river runs over a bed of poorly sorted material (e.g. glacial outwash plains). This circumstance counteracts deep erosion and favours the development of broad transverse sections.

A further noteworthy point is a difference between the morphologically effective scouring velocity for a sloping bank and for the river bottom. On the bottom a moderate movement of single grains does not in general produce any appreciable change in the surface there, since the arrival of other particles from upstream can replace the loss. But the situation is different as regards the sides. Grains which are set in motion as bed load usually have a component of motion towards deeper parts of the river bed. Consequently the eroded material cannot be replaced by other material from upstream to the same extent. Even a quite moderate transport of material may therefore in time lead to a considerable change in the shape of the river channel.

The morphological consequences of the above qualitative reasoning may be stated as follows. Factors encouraging the development of broad, shallow river channels are: the presence of easily eroded unsorted material, active transportation of bed load, and the instability of frictional material on sloping banks.

When there is a very strong transportation of bed load a broad, flat channel may provide the conditions for a temporary equilibrium. But the pattern of bed load transport does not continue unmodified. Sediment accumulates in banks or bars, the river channel is altered, and lateral erosion begins. The banks of sediment grow to elongated islets, and the river channel splits up. A system of sub-channels that diverge and converge again is formed—a *braided river* (cf. HJULSTRÖM 1953, and SUNDBORG 1954 a).

When there is moderate transport of bed load the equilibrium profile is deeper. There is a tendency for bars to form. Where sand banks are formed on the inside of bends in the river the transverse section becomes asymmetric, the flow becomes still swifter on the outside of the bend, and lateral erosion sets in. Secondary flow and other flow phenomena contribute to the development of a *meandering river*. Lateral erosion in a meander loop gradually slackens off until the radius of curvature becomes so large, and the asymmetry of the channel thereby so slight, that the velocity of flow along the outer sloping bank is near the critical value for erosion.

When there is little or no transport of bed load the longitudinal and transverse profiles become adapted to the transport and deposition of suspended sediment. A characteristic of the river channel is that it is deeper than in rivers where there is strong or moderate transport of bed load. The banks may become very steep if they are of cohesive material, since the critical velocity for erosion is then independent of the slope. The stability of the banks is not only a matter of resistance to erosion, but also of the tendency for slides to develop. In this case the shearing resistance of the soil is the factor which, in conjunction with the resistance to erosion and the magnitude of the suspended load, determines the shape of the transverse section. Meandering is common, but the tendency is not so pronounced as in sandy material, since the inner side of a curve is not built out by bars of bed load, and filling in with fine, suspended material generally is a slower process.

THE MOVEMENT OF BED LOAD

Definitions and types of movement

In previous sections there have been many references to bed load and suspended load, and the meaning of these terms has been assumed to be known. But for a more detailed discussion of bed-load transport and its various forms it is desirable to clarify the significance of the terms used. The definitions adopted are those recommended by the Subcommittee on Sediment Terminology.[1]

[1] Report of the Subcommittee on Sediment Terminology, Trans. Am. Geophys. Union, Vol. 28, p. 936, 1947.

"Contact load is the material rolled or slid along the bed in substantially continuous contact with the bed" ...

Contact load thus includes both *sliding* and *rolling* particles. Sliding is rather unusual in a river or stream, whereas rolling is the normal manner of transport for particles moving in contact with the bottom (see HJULSTRÖM 1939, p. 13).

"Saltation load is the material bouncing along the bed, or moved, directly or indirectly, by the impact of the bouncing particles" ...

Saltation load has been interpreted differently by different authors. GILBERT (1914, p. 27) described material in saltation as a cloud of particles occupying "a space with a definite upper limit" at the bottom of the current. This conception embraced all transport within a thin layer just above the bottom, and saltation in this sense is a very common type of movement (HJULSTRÖM 1939, p. 14). But with the narrower definition above saltation is a very unusual form of movement in water (KALINSKE 1942, p. 639). In air it is a different matter, since the particles attain much higher velocities, and the loss of weight due to buoyancy is negligible. The saltation load is therefore a considerable part of the total load transported by air.

"Suspended load ist the material moving in suspension in a fluid, being kept up by the upward components of the turbulent currents or by colloidal suspension" ...

Suspended load according to this definition includes the greater part of the material which in GILBERT'S sense is saltation load. The coarser fractions of suspended load may also be expected to "bounce" near the bottom, since turbulence leads to a fluctuating vertical component of flow, which instantaneously may have the same value as the particles' settling velocity. The boundary between saltation load and suspended load is rather diffuse, however.

"Bed load may be used to designate coarse material moving on or near the bed" ...

It should be noted that with this definition of *bed load,* saltation load and the suspended load near the bed are included in bed load. Strictly speaking, it is accordingly inexact to distinguish between bed load and suspended load. The correct classification according to the above definitions is instead: *contact load, saltation load, and suspended load,* or *bed load and non-bed load,* with the latter term covering suspended material that is lifted above the lowest layer of the stream.

When transport of bed load and transportation of suspended load are treated separately in the following sections, it should be borne in mind that these terms overlap to a certain extent. However, this overlapping is unimportant as long as we are not dealing quantitatively with amounts of transported material.

Surface features associated with transport of bed load

Another characteristic of the transport of bed load besides the manner in which the individual particles move is the special surface forms that appear where bed load moves along the bottom. Since these surface features, *e.g.* ripple marks, have often been preserved on exposed bedding surfaces of old sedimentary deposits and sedimentary rocks, study

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of them has provided an excellent method for attempting to determine the environment in which a particular sediment was deposited.

The literature on ripples and similar formations is very comprehensive. Good surveys of the field are to be found in KINDLE 1917, BUCHER 1919, KINDLE and BUCHER 1932, and SHROCK 1948.

In spite of the attention paid to ripples, there are still many unsolved problems. And in spite of many attempts it has not yet been possible to give a clear account of the basic causes of these surface features. This state of affairs must be attributed mainly to the complexity of the physical phenomena, but also to the difficulty of observing the formation process in the natural environment. Laboratory experiments have led to a convincing general picture of the surface forms and the way in which they arise. But such experiments are generally on a small scale, with small depths of water, and the results are therefore not directly applicable to natural watercourses.

The classical experiments by GILBERT (1914), and subsequent experiments by, among others, KRAMER (1932), CASEY (1935), and USWES (1935) have led to a distinction between the following four modes of transportation.

1. The *smooth mode of transportation* is characteristic of the movement when single particles have been set in motion.

2. Next follows the formation of *current ripples*, which are asymmetric transverse ridges with the steeper side downstream (fig. 19). The wavelength of ripples is usually between 1.5 and 30 cm (HJULSTRÖM 1939, p. 16), but may be larger. The ripples move downstream owing to scouring on the upstream side and deposition on the downstream side.

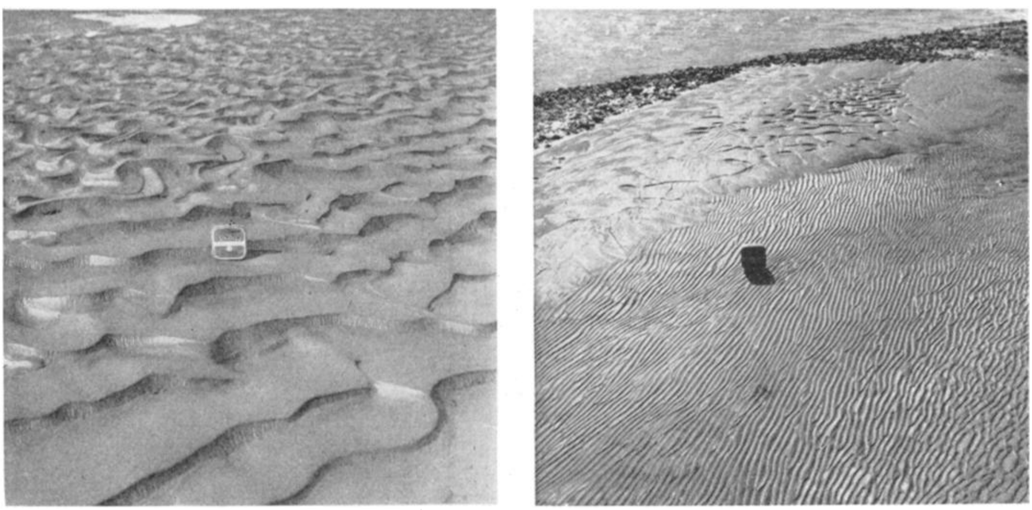

Fig. 19. Two types of ripples. To the left normal asymmetric ripples with steep downstream sides. The rippled surface has been preserved by an overlying thin lamina of silt. To the right a rippled sediment surface with small longitudinal ripples, developed as oscillation ripples near the bank-line of a river at rapidly falling river stage. The flow direction was from the upper right to the lower left in the photo (both photos from the outwash plain Hoffellssandur in Iceland 1951).

3. Still greater flow velocities destroy ripples, and material is then moved along an even surface by *sheet transportation.*

4. The fourth stage is movement in *antidunes,* i.e. low symmetrical sand waves which move upstream owing to scouring on the downstream side and deposition on the upstream side. Antidunes may have wavelengths of several metres.

MENARD (1950) has put forward modifications of the current conception of the modes of transportation on some points, on the basis of his own flume experiments and a critical examination of earlier investigations. One important observation is (p. 155), "that the velocity at which ripples are formed is the same as the competent velocity of the bed material. Even if only a few isolated sand grains in the flume are moving, in time they will form clumps. The current directly downstream from a clump impinges on the bed and causes a scour which forms a ripple. Sand movement becomes more rapid after the clumps are produced, and the last few ripples on a bed are formed much more quickly than the first ones."

The importance of this observation is that it reduces the four stages of transportation to three. The smooth mode of transportation is not an independent mode. As soon as movement begins no further increase in the flow velocity is required for the formation of ripples. It is only a question of time until particles begin to gather in clumps. As soon as this has occurred ripples quickly develop.[1]

MENARD (p. 159) also states velocities at which ripples may occur. As already mentioned, the lowest velocity is the competent velocity. He gives the highest velocity as that at which the flow ceases to be tranquil and becomes shooting (or torrential). "Ripples are destroyed at a current velocity which probably corresponds to a change in the nature of the flow from tranquil to torrential. The greater the water depth, therefore, the greater the current velocity required to destroy ripples." From the latter conclusion it follows that "antidunes and sheet transportation occur only if the flow is torrential".

MENARD'S work has helped considerably to clarify concepts. But many question marks still remain. We will consider some of them below.

From investigations in the valley of Klarälven and in several other rivers the author has observed that large transverse ridges or dunes, usually 2—20 m apart and with a distal slope 0.05—0.5 m high, are an important transport form (see fig. 20 and fig. 45). These ridges have a flat upstream side with a normal slope of about 1°, and a steep downstream side at roughly the angle of repose. In the sequel ridges of this type will be referred to as *bars,* although in the literature there is often no distinction between ripples and bars.[2]

Bars may be covered with ordinary current ripples. One may sometimes observe a certain simultaneous movement of both ripples and bars, but usually the bars are the typical transport forms for high water-stages and a swift current. Ripples are destroyed at high flow velocities, and sand is then moved in what the author would describe as

[1] MENARD does however suppose that the smooth mode of transportation may occur for relatively coarse material, between 1 and 4 mm.

[2] There are several descriptions of similar formations in the literature. Cf., for instance, DE GEER 1911, p. 76, KINDLE and BUCHER 1932, p. 647, FORCHHEIMER 1930, p. 539.

Fig. 20. To the left transverse bars in the river Klarälven, exposed at low stage. The flow was from the left to the right in the photo (Transtrand in Dalby parish 1953). To the right nearly symmetric ripples in very fine sand and silt. The flow was from the left to the right (Munkebol in Norra Ny parish 1953).

sheet transportation on the upstream sides of the bars and deposited on the downstream sides, which gives rise to the well-known stratigraphic feature of *cross-bedding*. If the velocity falls ripples may form over the bars, which then remain practically stationary. Both ripples and bars are associated with flow well on the tranquil side of the boundary between tranquil and shooting flow.

In the experience of the author there are no definite forms intermediate between ripples and bars, and the manner of formation seems to be quite different in the two cases. It would be more correct to draw a clear distinction between ripples and bars. Such a distinction would imply that a change in the type of flow is not a necessary condition for the destruction of ripples. The change would instead be due merely to an increase in the flow velocity. Also, sheet transportation would not be confined to the shooting regime of flow.

Another phenomenon not included in the scheme of surface forms is the occurrence of symmetrical ripples, often of rather short wavelength, in fine sediment (less than 0.2 mm). Such ripples, with a wavelength of 10—30 cm, are sometimes observable in the fine glacio-fluvial deposits in the valley of Klarälven (see fig. 20). It was at first tempting to interpret these as a conform deposit of finer material on top of normal ripples that had formed in a sandy sediment. Closer examination revealed, however, that although this interpretation is often correct, symmetrical ripples may also be independent formations, having a different wavelength from the underlying ripples.

These symmetrical ripples are formed at depths of some tens of metres, where the flow must certainly have been tranquil. Hence they are not antidunes in the accepted sense. Nor are they oscillation ripples, being rather ordinary ripples with the difference that the profile is symmetric.[1] As far as the author is aware, the environment and process of their formation have not previously been mentioned in the literature.

It is undoubtedly impossible to give a satisfactory explanation of the various transport forms except after comprehensive laboratory and field investigations, combined with a thorough theoretical analysis. Such investigations must be planned on the basis of some reasonable working hypothesis if the results are to be significant, and a relatively detailed discussion of some viewpoints may therefore be of value even at this early stage. Some views of the author on the problem of ripples are here put forward, with the express reservation that *they are not intended as a developed theory, but only as a working hypothesis for further studies.*

Perhaps the most rational approach to the study of surface forms associated with transport of bed load is a development of the extensive investigations carried out by SHIELDS (1936). He found that the surface forms were greatly dependent on the ratio of the grain size to the thickness of the laminar sublayer (p. 17). When the laminar sublayer was well developed, fine ripples were obtained of relatively great height in proportion to the length. For coarser grains, on the other hand, where there was no laminar sublayer or where it was disturbed by large projecting particles, there formed what SHIELDS called "*Schuppen*" or "*Bänke*".

When the flow is smooth or transitional there exists a laminar sublayer, either fully or partially developed. As long as other irregularities than those due to the uniformly distributed grains are absent, and as long as there is no movement of particles, the laminar sublayer is of the same thickness at every point of the bed (supposing the flow to be two-dimensional). As soon as sand grains begin to move the bed surface becomes slightly uneven, since the effect of random erosion and transportation cannot be the same at every point. As MENARD has pointed out, "if only a few isolated sand grains in the flume are moving, in time they will form regular clumps".

The slight irregularities of the bed will now affect the laminar sublayer and the flow in general. Irregularities that project into the sublayer or even through it will "drag down" water of greater velocity towards the bed behind the elevation. The effect is a thinning out of the laminar sublayer behind the elevation, and possibly a complete collapse of the layer. The shearing stress increases, and scouring begins. This is quite in accord with the observations of MENARD.

In this connection reference may also be made to two interesting laboratory investigations by SCHLICHTING (1936) and JACOBS (1938), of the flow behind obstacles. One of their observations was that an obstacle gave rise to a negative "shadow effect" in that the heightened turbulence behind the obstacle brought water with a higher flow velocity nearer the bottom a little downstream from the obstacle.

Once the initial equilibrium is disturbed the process continues at an accelerating pace. It should be noted that the bed surface *downstream* from a scour must be convex to the

[1] It may be noted that these ripples are not the same as the so-called meta- and para-ripples described by KINDLE and BUCHER (1932, p. 649).

stream. The thickness of the laminar sublayer should increase as the flow passes over such a region, with a consequent tendency to separation of flow. In its turn this gives rise to a local decrease in the transport and erosion capacity, with consequent accumulation of sediment. The accumulation continues until the convexity is so pronounced that separation of flow is definitely established. The point of separation will then coincide with the distal slope of an incipient ripple (cf. fig. 6 and fig. 7).

The ripple, once formed, is itself an obstacle which gives rise to further scouring and accumulation downstream from it. A series of ripples forms. When the flow is tranquil the formation of ripples may also affect the flow upstream, so that the rippled area may conceivably spread out even in the upstream direction from the point of initiation. The oscillations of the sublayer naturally also affect the flow in the transverse direction, and so extensive rippled areas may develop.

If this view is correct, one may draw the conclusion that *an even surface is an unstable transport form.* Each little irregularity on a surface that is smooth in the practical sense then inevitably leads to an accelerating rippling process. Accordingly, *the only stable surface is a rippled one when the flow is smooth or transitional.*

As regards the important problem of wavelength and amplitude, it must be presumed to be connected with the properties of the main flow (velocity and state of turbulence), the thickness of the laminar sublayer, the shape of the existing surface, and the grain size of the bed material. The primary question concerns how far behind an obstacle the disturbed flow impinges on the bed. It is important to note that in the present interpretation the formation of ripples is a boundary layer phenomenon, which is hardly affected by the total water depth, unless this is so small that the "wave motion" extends throughout the greater part of the stream.

According to the above hypothesis asymmetric ripples of normal type should occur whenever the velocity is sufficiently large for transport, if the grain size is suitable. The material should not be so fine that it goes into suspension, nor so coarse that the flow is rough. These limitations agree very well with the empirical limits of velocity and grain size mentioned by MENARD (cf. MENARD 1950, p. 154, and fig. 23 of the present publication).

With large grains or higher velocities the limit at which the flow becomes completely rough is soon reached. The turbulent flow then acts on every particle in an exposed position. The conditions are then inappropriate for an alternation of scouring and accumulation such as is associated with a variation in the thickness of the laminar sublayer. The speed of the dislodged particles is increased, and with increasing frequency they are carried from the crest of one ripple to that of the next, or pass over the crests of several ripples without coming to rest. The regular deposition of material on the distal sides of ripples ceases, and the ripples are erased. The stage is then attained which is usually referred to as sheet transportation. The flow is still tranquil, unless the water is very shallow (cf. fig. 1).

It would perhaps be expected that the bed would be devoid of irregularities when the ripples had been destroyed. However, a line of reasoning analogous to that applied to the laminar sublayer should apply in this new case, although the flow is now completely turbulent. At convexities in the bed there should be a tendency to separation of the turbulent layer on the downstream side, with lessening of the transport capacity and accumulation in consequence. As the accumulation proceeds there comes a point where separation is accomplished, and a distal slope forms there. This corresponds to the development of what has been termed a bar.

The important difference compared with ripple formation is that when bars are formed the whole of the turbulent boundary layer, i.e. the whole depth of water, may affect the process. The evolution of transverse bars may be expected to depend on the depth of water. Where the flow on the upstream side of a bar is up a slight slope the velocity and the tractive force increase continuously, and the flow is able to carry the increasing bed load (cf. p. 213). But it is obvious that the bar cannot rise above a certain limit, unless the flow is divergent. Accordingly, the surface of the bar should flatten out towards the horizontal farther downstream. But when the surface approaches the horizontal the possibility of separation increases, and in general the bar will not attain more than a certain height and length. This explanation accounts for the fact that the surface of bars is often slightly convex, as well as the fact that smaller bars are sometimes observed superposed on the larger (cf. the descriptive section, p. 271, and fig. 44).

According to this hypothesis the formation of bars is a process which differs from the formation of ripples in that it is affected by the geometry of the watercourse. The size and shape of bars should depend on the dimensions of the stream, especially its depth, and a gradation of forms may be expected, from small transverse bars to large delta lobes.

As soon as the velocity becomes so large that the flow is shooting, all transverse elevations are destroyed, since irregularities such as ripples and bars, which are built up of easily moved material could not exist at the hydraulic jump which must occur if the distal slope were to persist. Long standing waves appear at a state of flow near the critical boundary between tranquil and shooting (cf. p. 137). These are associated with so-called antidunes. In very shallow streams a direct transition from ripples to sheet transportation and antidunes may be expected.

As regards the symmetrical ripples mentioned previously, one can only surmise that the process of deposition is in such cases affected by a wave movement in the bottom layer of water. Since the sediment is so fine that it would not be transported along the bed, the conditions for the formation of a distal declivity are not fulfilled, and the shape is instead symmetric. It is conceivable that the wave movement arises in a manner analogous to that proposed for the formation of asymmetric ripples. It is also conceivable, and even probable, that the pronounced density stratification which must have occurred in such cases gave rise to a standing wave motion in the layers near the bottom (cf. HJULSTRÖM 1935, pp. 333—4, and LONG 1953, 1954, and 1955.)

Finally, it may be mentioned once more that the above reasoning is not intended as a definitive explanation of the process of formation for ripples and bars, but merely as a contribution to discussion on the subject which may direct attention to some factors worthy of further study.

Quantitative calculation of the transport of bed load

In most rivers the material transported as bed load is supposed to comprise a small percentage of the total sediment transported. However, the percentage varies considerably with local conditions, particularly with the grain sizes composing the bed, and with the amount and texture of the suspended material brought down by the stream. According to LANE and BORLAND (1951), it may be supposed that bed-load transport, where it occurs, varies between 2 and 150 % of the amount of suspended material, i.e. between about 2 and 60 % of the total transport. The normal percentage appears to be of the order of 10 %.

Material transported as bed load has a more immediate effect on the configuration of the river bottom than suspended material has, and it is therefore on balance of equal importance from the geomorphological point of view. Transport of bed load is also important from the technical point of view, a factor to be taken into account when planning regime channels or water reservoirs, for instance.

In general it is no difficult matter to measure transport of suspended material directly by means of water samples, but hitherto no successful bed-load sampler has been constructed which provides reliable measurements of bed-load transport. Unless this transport can be determined indirectly, it is thus necessary to make use of empirical or theoretical deduced *bed-load formulas.*

Many bed-load formulas have been put forward. Several of them can only be applied to the conditions under which they were derived, and are not generally usable.

Important publications prior to 1940 are those of SCHOKLITSCH (1914), SCHAFFERNAK (1922), MACDOUGALL (1933), MEYER-PETER (1934), STRAUB (1934), USWES (1935), O'BRIEN (1936), SHIELDS (1936), and CHANG (1939). That of SHIELDS is especially worthy of note because of the dimensional homogeneity of the formulas, and the circumstance that the density of the particles has been taken into account.

MEYER-PETER and MÜLLER have subsequently (1948) published a dimensionally correct bed-load formula, which is based on a large number of experiments and seems to give good results.

The more complete understanding of turbulent flow achieved in recent years has led to two bed-load formulas which are widely used in the U.S.A. These are KALINSKE's bed-load formula (1947), and EINSTEIN's formula (1950). EINSTEIN's method has subsequently been developed by SCHROEDER and HEMBREE (1956) to calculate the total transport of sediment.

A more detailed description of the various methods may be obtained from the original papers or from current textbooks, e.g. LINSLEY, KOHLER, and PAULHUS (1949, p. 336), or ROUSE (1950, p. 794). Briefer commentaries are to be found in a report on "Recent developments in the hydraulics of open channel flow" (Proc. of Inst. of Civ. Eng., Part III, Vol. 4, Dec. 1955, no. 3, p. 1027), and in CARLSON and MILLER (1956, pp. 953—8).

The sources of error involved in quantitative calculations of bed-load transport are still rather large, and it is not yet possible to give a definite preference to any one of the modern methods.

For the author's investigations in the valley of Klarälven (see p. 303) KALINSKE'S formulas have been used. The reason for this is that the assumptions underlying KALINSKE'S formulas are closely similar to the assumptions used in deducing the curves for the critical erosion velocity in the present publication. It is not possible to state with any exactness the accuracy of the calculations, but it does at least seem that the trend in the variation of the bed load, and its order of magnitude, are correct.

KALINSKE'S equation may be written

$$G = \frac{2\gamma_1\, k\, \varrho_m}{3} \sqrt{\frac{\tau_o}{\varrho_w}}\, \Psi\left(\frac{\tau_c}{\tau_o}\right) \tag{50}$$

where G = the transport per unit width of the stream, expressed as a weight per unit time $(g \cdot cm^{-1} \cdot sec^{-1})$

τ_c = the critical value of the bottom shear, *i.e.* the value at which transport commences

τ_o = the bottom shear

$\Psi\left(\frac{\tau_c}{\tau_o}\right)$ = a special function for bed-load transport, defined and presented in diagram form in KALINSKE'S paper. The function is dependent on the state of turbulence.

Other symbols in the equation have the same meaning as previously in the present publication.

THE MOVEMENT OF SUSPENDED LOAD

The vertical distribution of suspended material

The suspended material in a stream is kept in suspension by the turbulent motion of the water. The basic concepts involved in turbulent exchange and something of its application to the transport of sediment were considered in the section on turbulent exchange (p. 152).

From that starting point we will now proceed to derive an expression for the concentration of sediment at various levels in a watercourse, on the assumption that equilibrium prevails. The derivation is closely similar to that first given by ROUSE (1938).

The condition that there is a state of equilibrium signifies that the amount of sediment falling by its own weight per unit time across an arbitrary horizontal unit surface is equal to the difference between the amount of sediment carried upward by the turbulent motion from lower layers of higher sediment concentration and the amount of sediment carried downward by the turbulent motion from upper layers of lower sediment concentration. This equilibrium condition may be expressed mathematically as follows:

$$\varepsilon_s \frac{ds}{dz} + cs = 0 \tag{51}$$

where s is the concentration of suspended material of a certain grain size, and c the settling velocity of the particles in water. ε_s is the transfer coefficient for sediment.

According to equation (35) on p. 156, ε_s may be written

$$\varepsilon_s = \frac{\beta\ \tau_0}{\varrho_w \frac{d\bar{u}}{dz}} \left(1 - \frac{z}{z_m}\right) \tag{52}$$

where β is a factor of proportionality (usually close to 1), and z_m is the height above the bottom at which the flow velocity is a maximum (in general approximately equal to the depth of water).

Differentiation of the equation for the vertical distribution of velocity gives

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\varkappa} \frac{1}{z} \tag{53}$$

This is substituted in (52), and the expression obtained is combined with (51). Simultaneous separation of the variables gives the differential equation

$$\frac{ds}{s} = -\frac{c\ dz}{\beta\, \varkappa\, u_*\, z \left(1 - \frac{z}{z_m}\right)} \tag{54}$$

Integration of this equation from a reference level a to the level z gives

$$\frac{s_z}{s_a} = \left(\frac{z_m - z}{z} \cdot \frac{a}{z_m - a}\right)^{\frac{c}{\beta \cdot \varkappa \cdot u_*}} \tag{55}$$

u_* may be obtained from eq. (16) or (18), remembering that $k_s = 30\, z_0$ (see p. 148). The expression obtained for u_* is inserted in (55). It is assumed that the flow velocity is greatest near the surface of the water, whereupon z_m may be taken as approximately equal to the depth of water, d. $\varkappa$ has the value 0.40 (p. 154, and β is taken to be 1 (p. 156). These formulas lead to the following formula for the vertical distribution of the suspended material:

$$\frac{s_z}{s_a} = \left(\frac{d - z}{z} \cdot \frac{a}{d - a}\right)^{\frac{6.25\, c \ln \frac{d}{z_0}}{u_{\max}}} \tag{56}$$

The settling velocity for small particles (radius r less than 0.05 mm) may be computed from STOKES' formula,

$$c = \frac{2}{9} r^2 \frac{\varrho_m - \varrho_w}{\mu} g \tag{57}$$

where ϱ_m and ϱ_w denote the densities of the sediment and the water, μ the coefficient of viscosity for water at the temperature concerned, and g the acceleration due to gravity.

For grain sizes above the limit of validity of STOKES' formula c may be determined with the aid of an empirically constructed diagram of the settling velocity as a function of "the sedimentation diameter" (see, for instance, RUBEY 1933, or BROWN 1950, p. 781).

From equation (56) it is possible to calculate the concentration of material of a particular grain size at any level in a river, provided the concentration at some reference level is known. One must know the depth of the water, the settling velocity of the particles, as well as the flow velocity and the roughness of the bottom.

Care should be taken in applying (56) to the concentration of sediment in a natural watercourse. In the immediate neighbourhood of the bottom the concentration should approach infinity, and near the water surface zero, according to the formula. This does not correspond to the real state of affairs, of course. The formula is therefore invalid for levels either near the bottom or near the water surface. The equation has been deduced on the assumption that a state of equilibrium prevails. In a river the velocity and depth vary from point to point along a longitudinal profile. At some places the river takes up sediment into suspension by erosion of the bed, at other places sediment is deposited. Consequently it cannot be expected that the concentration of suspended material in a particular place should exactly agree with the equilibrium value.

Nevertheless, measurements in rivers have shown that eq. (56) is a good representation of the actual conditions (CHRISTIANSEN 1935, RICHARDSON 1937, ANDERSON 1942, cf. also BENDER 1956). Some divergences occur, however. The numerical value of the exponent in eq. (56), as determined from laboratory experiments, may differ somewhat from that theoretically deduced, mainly due to the deviation of the value of β from 1. If the content of sediment is so high that density stratification occurs, the turbulent transfer coefficient is affected, as mentioned on p. 158. In this case the equation is not valid, as experiments by VANONI (1953), among others, have also shown.

Recapitulating, it may be stated that *formula (56) is on the whole a correct representation of the concentration of sediment in sections of a river where equilibrium prevails, and where there is no density stratification.*

As examples of the distribution of suspended material in a river the ratio s_z/s_a, *i.e.* the ratio of the concentration at a level z to that at a reference level a, has been calculated for various sets of conditions. The results are shown in figs. 21 and 22.

The various conditions used in the calculation are as follows.

1. The roughness parameter z_0 has been taken to be 0.2 cm. This corresponds to an equivalent sand roughness of 6 cm. The calculations are thus valid for a relatively flat bed of pebbles 6 cm in diameter, or for a bed composed of finer material but with the given degree of roughness on account of ripples or bars. According to the author's investigations in Klarälven the degree of roughness assumed is fairly representative of sandy beds with ripples or bars there (p. 284).

2. The reference level a has been taken 6 cm above the bottom, i.e. a is the same as the equivalent sand roughness.

3. The concentration of sediment has been calculated for four velocities: 25, 50, 100, 200 cm/sec. The respective diagrams are on the left in fig. 21, on the right in the same figure, on the left in fig. 22, and on the right in that figure. The velocity is the maximum velocity, i.e. approximately the velocity at the surface.

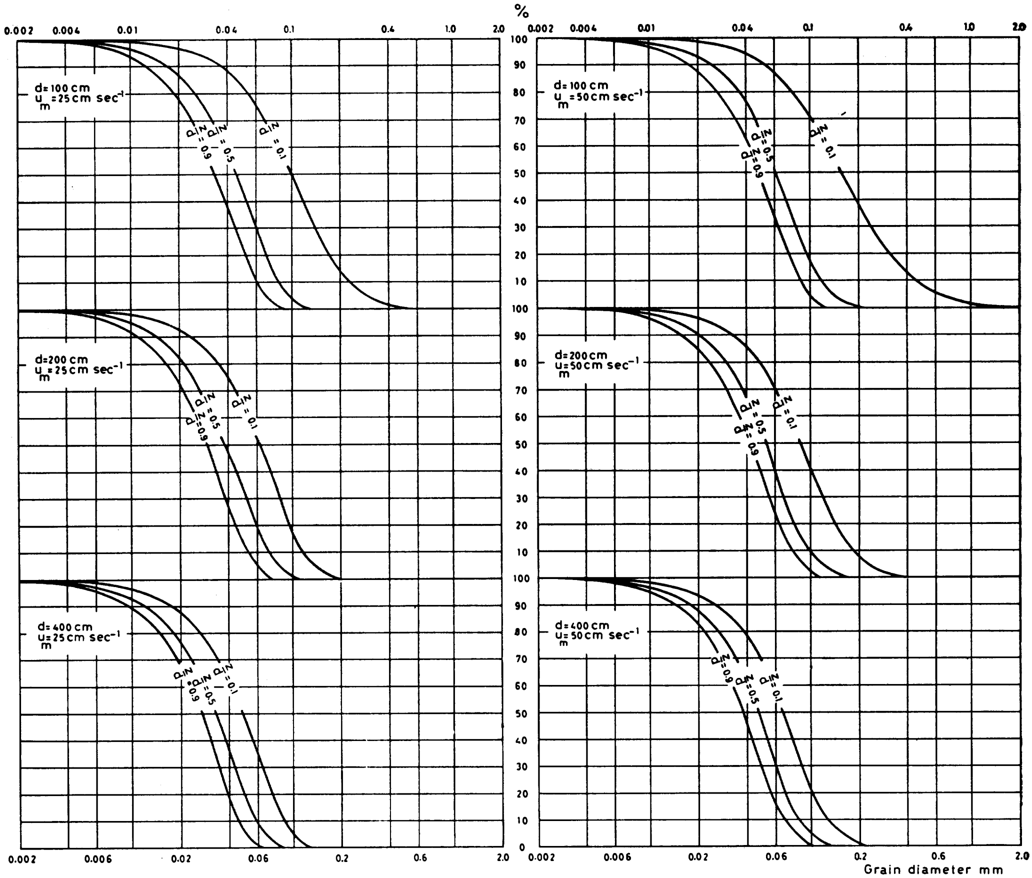

Fig. 21. The distribution of suspended material in a river for different depths d and surface velocities u_m. calculated with formula (56). The assumptions involved in the calculations are stated in the text. The curves gives the ratio in per cent of the concentration at a level z to that at a reference level a (6 cm above the bottom) for different grain sizes. The three curves in each diagram refer to three values of the ratio z/d (d = the water depth).

4. The calculations have been carried out for three depths of water in each case: 1, 2, and 4 m. The respective diagrams are at the top, in the middle, and at the bottom of each figure.

5. The calculations have been carried out for three values of z/d: 0.1, 0.5, and 0.9. This means that for water 1 m deep the concentration has been calculated for the levels 0.1, 0.5, and 0.9 m above the bottom, in the case of 2 m depth for the levels 0.2, 1.0, and 1.8 m, and in the case of 4 m depth for the levels 0.4, 2.0, and 3.6 m, in each case as a percentage of the concentration at the reference level 6 cm above the bottom.

6. The settling velocity c has been calculated from STOKES' formula for particles less than 0.1 mm in diameter, the water temperature being taken as 15° C, $\mu = 1.14 \cdot 10^{-2}$ $cm^{-1} \cdot g \cdot sec.^{-1}$, and $\varrho_m = 2.65$ g cm^{-3}. The calculations apply primarily to round quartz

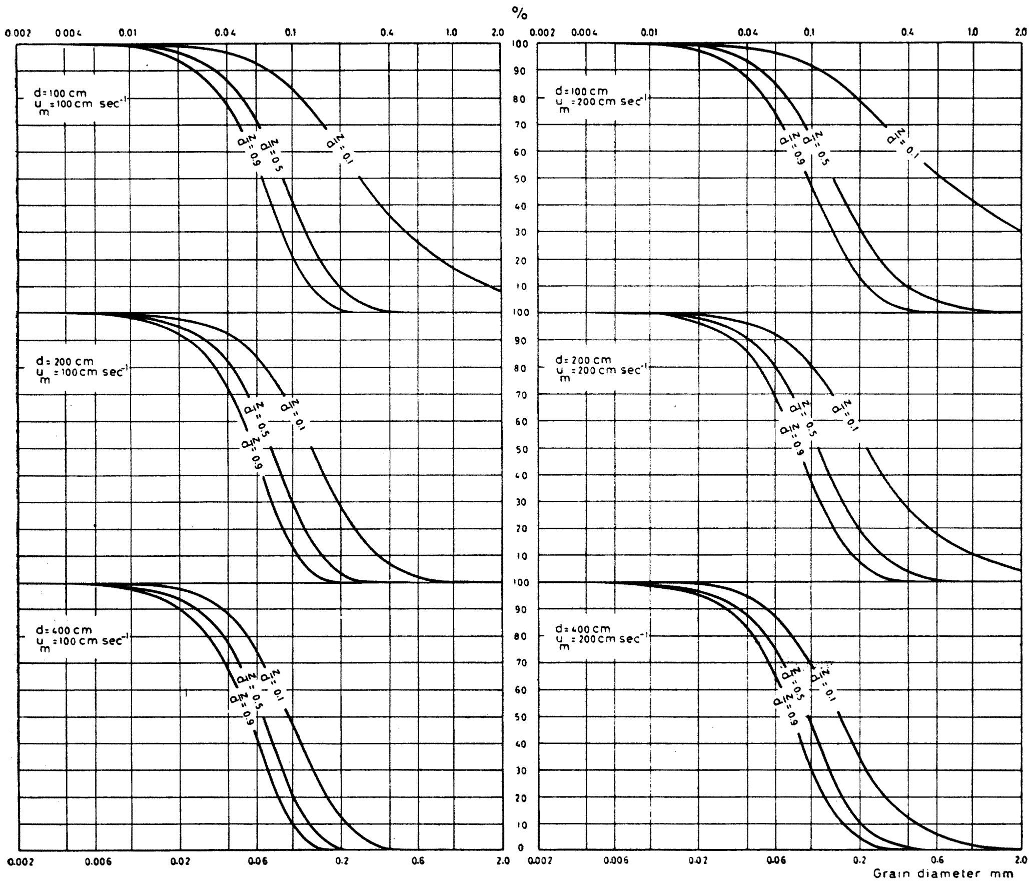

Fig. 22. The same as fig. 21, but with other flow velocities.

grains, but may of course be applied to grains of a different density and shape if the sedimentation diameter is known. Empirical values of c have been used for larger grain diameters. (BROWN 1950, p. 781.)

From the diagrams it is possible to see directly how large the average concentration of sediment may be expected to be for an arbitrary particle size at any one of the three levels considered, and for different velocities and depths. The diagrams more or less speak for themselves, but a few comments may not be out of place.

The concentration of grain sizes less than 0.01 mm is almost the same at all levels. Grain sizes above 0.4 mm occur only near the bottom. As the velocity increases the curves are displaced to the right, which means that coarser grains are taken up into suspension. The displacement of the curves to the right is most pronounced for the coarser fractions of sediment. This is due to the fact that the settling velocity for particles larger than 0.1 mm does not increase as rapidly as the square of the radius, as it does within the range of validity of STOKES' formula.

The boundary between bed load and suspended material

In connection with figs. 21 and 22 it seems appropriate to discuss the question of the boundary between transportation as bed load and transportation as suspended material. Is it possible to decide whether a sediment has been transported as bed load or in suspension merely from a knowledge of the grain size?

As just mentioned, the curves clearly show that increasingly coarser material may be taken up into suspension as the flow velocity increases or the depth of water decreases. The boundary depends on the flow conditions. Also soil analyses have shown (cf. p. 277) that a deposited material which was transported in suspension may be coarser than a sediment which was certainly transported as bed load.

An important factor to be borne in mind when considering transport and deposition of suspended material is the nature of the river bed. If the bed does not contain the same grain sizes as those found in suspension above it, this must signify that all the suspended material is carried past without there being any appreciable interchange with the material of the bed. If, on the other hand, the river bed is made up of the same type of sediment as that carried by the current—what is known as an *alluvial reach* (MACKIN 1948)—there is a continual exchange of particles between the river bed and the water. The particles bounce along near the bottom, sometimes in suspension, sometimes as bed load, and sometimes at rest (cf. p. 205). In a state of equilibrium the number of particles picked up from the bed is equal to the number deposited, for a particular area and time.

In accordance with a suggestion by INMAN (1949, p. 60) that the curves for the critical erosion velocity may be combined with the calculated curves for the concentration of suspended material, we can determine from figs. 21 and 22 the grain sizes for which the concentrations at the 10 cm level are respectively 10, 20, 30, 40, and 50 percent of the

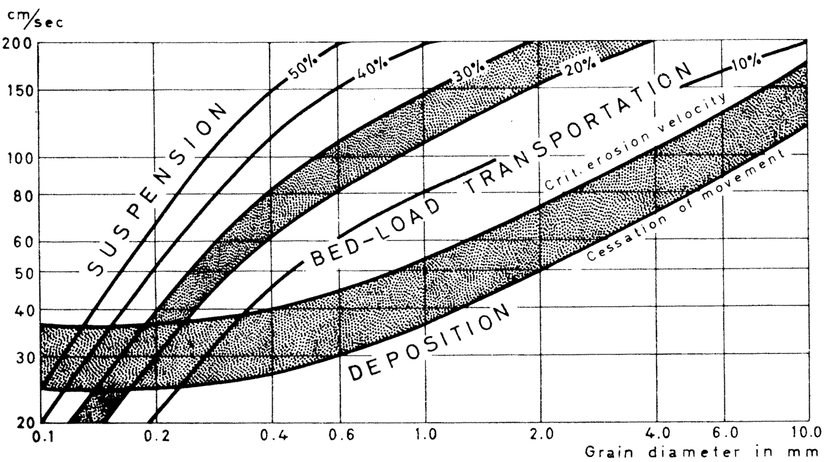

Fig. 23. The relation between flow velocity, grain size and state of sediment movement in uniform material of density 2.65 g cm^{-3}. The flow velocities are those 1.0 m above the bottom.

concentration at the reference level, 6 cm. The calculation is for a water depth of 1 m. The values obtained are plotted in a diagram, where curves for the critical erosion velocity and the velocity of cessation of movement are also included (fig. 23).

An important question now is: how large should the concentration at a certain level be in relation to the concentration at a reference level near the bottom in order that the material may be said to be in suspension? LANE and KALINSKE (1939, p. 640) have observed that "sand sizes giving a value of t_c of one or greater would not be found in suspension in any greater quantity". t_c is a symbol for c/u_*.

If c/u_* is put equal to one, the corresponding curve in our family of curves is that for a 25 % concentration. The region for s_z/s_a between 20 and 30 % has been dotted to indicate that this interval is the somewhat diffuse boundary for material that may occur in suspension to any considerable extent.

The curves in fig. 23 divide up the diagram into different regions. For flow velocities less than the velocity for the cessation of movement sediment can no longer be moved along the bottom. Deposition occurs instead. Within the dotted band between the velocity for the cessation of movement and the critical erosion velocity there is transport of bed load but no real erosion. In the region between the two dotted bands there is transport of bed load and possibly erosion, but the amount of suspended material of the grain sizes under consideration is small. Above and to the left of the upper dotted band the amount of suspended material increases. Farthest down on the left of the diagram there is a region where neither erosion nor transport of bed load can occur, but where transport and deposition of suspended material may be expected.

The size of the *finest* material transportable as bed load may be obtained from the intersection of the two dotted bands. It will be seen that it is between 0.15 and 0.20 mm, which is in good agreement with the result obtained by INMAN (1949, p. 57), 0.18 mm. *It should be stressed that this limit is diffuse, and in no way excludes the possibility that coarser material may occur in suspension when the flow conditions are favourable.*

THE DEPOSITION OF SUSPENDED MATERIAL

Aggradation of a river bed, the building up of levees, layers of mud and silt left behind in flooded areas, and the filling in of lake basins are all intimately connected with the deposition of suspended material. The morphological importance of the deposition process is thus obvious. A common hydrotechnical problem is the prediction of how suspended material will be deposited in a planned reservoir or dam basin. Large sums are involved when a reservoir is to be planned so that it can accommodate deposited sediment without undesirable reduction of the reservoir's useful life.

In spite of the fact that the mechanism of transport of suspended material is relatively well understood, "methods of predicting the distribution of the sediment that is to be deposited in a reservoir are still in need of improvement" (CARLSON and MILLER 1956, p. 953—19).

There are various factors that affect sedimentation in a river or lake. Some of the most important are: the amount of suspended material in the stream and its size distribution, the configuration of the river or basin, the course of the stream in the sedimentation region—including possible occurrence of density stratification—the water discharge in relation to the storage capacity, the "capturing effect" of vegetation, flocculation of colloidal suspended material, and so on.

In attempting to assign an explicit mathematical form to some of these factors the author has adopted the following simplifying assumptions.

1. The ratio of the number of particles in a certain interval of grain size that are picked up from the bed to the number that are deposited, is denoted by φ (c). φ (c) is an unknown function of, among other quantities, the grain size (or the settling velocity c), and flow parameters. However, it is reasonable to assume that φ (c) is certainly zero when the flow velocity is less than the velocity for the cessation of movement (cf. fig. 23). The formation of a laminar sublayer is probably of essential importance if particles are to be left undisturbed once they have been deposited. Vegetation growing on the bottom, or vegetation in a flooded area, reduces the velocity of flow near the bottom, and therefore helps to lower φ (c).

2. The amount of material in a particular size fraction which falls on to unit area of the bottom per unit time may be expressed quantitatively as $c \cdot s_b$, where c is the settling velocity for the grain-size fraction concerned, and s_b is the concentration of this fraction in the water near the bottom.

3. We assume that there is a stationary state or state of equilibrium in the body of water under consideration as regards the flow of water and concentrations of sediment, so that the concentration of sediment at any point is practically constant with time: the net deposition of sediment of a particular grain size on an area Y during the time dt is given by $Y \cdot c \cdot s_b\,[1-\varphi(c)]\,dt$. The total net sedimentation is obtained by integrating over all grain sizes.

4. We also assume that changes in the state of flow along the direction of flow do not take place so rapidly that eq. (56) cannot be used to determine the vertical distribution of suspended material in any position.

5. The average concentration[1] of material in a particular fraction is denoted by s_m. The concentration at the bottom, s_b, is taken to be equal to $\Phi(c).\ s_m$, where $\Phi(c)$ is a function which, like $\varphi(c)$, depends on the settling velocity and the flow conditions. For given flow conditions $\Phi(c)$ may be obtained graphically from curves of the type shown in figs. 21 and 22.

6. Deposition of sediment will lead to a decrease in the concentration of sediment in the downstream direction, unless local erosion or tributary streams supply more sediment. The total amount of material in the fraction considered which passes per unit time a vertical area of unit breadth and of the same depth d as the water is $u_m \cdot d \cdot s_m$, where u_m is the mean velocity, averaged over the vertical distribution.

[1] The average concentration is the average concentration in the volume of water passing per unit time an area which is perpendicular to the flow, of unit breadth, and of the same depth as the water.

Let us now consider two-dimensional flow with constant d and u_m, and with a system of coordinates that moves with the flow, i.e. with a velocity u_m with respect to the bottom. Then we may write the differential equation

$$c \cdot s_m \cdot \Phi(c) \; [1-\varphi(c)] \, dt + d \cdot ds_m = 0 \tag{58}$$

Since d and u_m are constant by assumption, $\Phi(c)$ is also constant. We will discuss the simple case where $\varphi(c) = 0$, i.e. where no material is picked up from the bottom.

Integration from time 0 to time t then gives

$$s_{m_t} = s_{m_0} \cdot e^{-\frac{c \cdot \Phi(c)}{d} t} \tag{59}$$

Eq. (59) gives the concentration of material of a particular grain-size fraction that remains in suspension after the water has flowed for a period t with the given velocity and depth of water.

In practice it would often be difficult to apply (59), principally because the flow conditions vary from point to point in a stream. The retardation of water near the bank lines and the separation of the flow from the shores where these diverge, are factors of very great importance for sedimentation.

So it would really be necessary to know the depth, flow velocity, and state of turbulence at each point of a watercourse in order to be able to analyse the process of sedimentation in detail. However, the mathematical treatment of turbulent diffusion and sedimentation in terms of a turbulence model would entail great, probably insurmountable, difficulties.

In making a sedimentation forecast it is therefore necessary to simplify the reality to a considerable degree. The problem then becomes a matter of choosing the simplifications so that they have the least possible effect on the reliability of the final result.

On the supposition that some sort of average values for depth and state of turbulence together with a relatively detailed knowledge of the flow configuration in a region under consideration may conceivably lead to satisfactory results from eq. (59), the author has worked out a method for predicting sedimentation of non-colloidal suspended material. The method is outlined on p. 232.

It may seem that the computation of sedimentation with the help of eq. (59) or some other formula is a matter of purely technical interest. But the possibility of estimating where and to what extent sediment will be deposited in an actual river or other body of water will of course improve our knowledge of the morphology and stratigraphy of sediments in relation to the environment in which they were laid down.

It may be noted that the well known logarithmic reduction of the thickness of a deposit in the direction of transportation finds a natural explanation from eq. (59). Likewise, the gradual decrease of the grain size in the direction of transportation is also apparent from the equation, as well as the distribution of a mineral with a settling velocity differing from that characteristic of the particular grain size, e.g. heavy minerals.

The author hopes to return to these questions on a later occasion, when more data have been gathered together for testing the assumptions underlying eq. (59).

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CHAPTER III

METHODS

The relative salt-dilution method applied to turbulent diffusion

Introduction

The flow velocity and state of turbulence determine the capacity of a stream to transport bed load and suspended load. As already pointed out (p. 151), it is therefore desirable to carry out systematic investigations of the small-scale turbulent fluctuations in various types of stream, with different bottom configurations and of different orders of size.

However, the extremely irregular conditions in a natural stream, with large variations of the velocity and intensity of turbulence from point to point, make it difficult to obtain from a determination of the state of flow at certain points a definite picture of the manner in which suspended matter spreads, for instance. The turbulence cannot be regarded as isotropic. The flow lags behind in the neighbourhood of the river bed. Furthermore, on account of "edge effects" due to the river shores the flow cannot be treated as two-dimensional, and the theoretical treatment of the spreading is therefore very complicated.

Consequently there is at least as great a need for investigations of turbulent mixing on a macro-scale. How does water spread out from a point in various directions downstream? What is the probability that water particles from one point will pass another point downstream from the first? What is the probable time of transit from one point to another in a river? How does the water in a lake or a bulge in a river move, and how does mixing between river water and stationary water in a basin proceed?

These and similar questions point to the need for a method of studying flow and turbulence conditions on a macro-scale. An obvious method is to use a modification of one of the electrochemical methods employed in some countries for water discharge measurements.

Electrochemical methods for water discharge measurements

Electrochemical methods for discharge measurements all involve the injection of brine in order to determine the flow rate in a closed conduit or a free channel. There are three main types of method (cf. Howe 1950, pp. 206—209):

1. *The salt-velocity method,* developed by Allen and Taylor (1923), "makes use of the increased electrical conductivity of salt water with respect to that of natural water to indicate the time required for a slug of brine to travel through a previously measured volume of conduit".

2. *The salt-dilution method* "involves adding brine of known concentration at a known rate to natural water of previously determined salt concentration but unknown flow rate".

3. *The electrical-conductivity method* "combines certain features of the two above described and . . . appears to avoid the disadvantages of both while retaining their advantages. The method . . . makes it possible to determine the degree of dilution by integrating a conductivity-time graph . . .".

Closely related to the electrochemical methods, and like them adaptable to turbulence measurements, are measurements with radioactive isotopes, which have been used in recent years to determine flow velocities and discharge rates in rivers and lakes (MONTENS 1954).

The author's method for determining turbulent mixing is an adaptation of one form of the electrical conductivity method. This way of measuring river discharge has been used in Norway, where it has been developed mainly by R. SÖGNEN at Norges Vassdragsvesen (AASTAD and SÖGNEN 1952). The method has been called *the relative salt-dilution method* (relative Verdünnungsmethode). Its main features are as follows:[1]

When a certain amount of a primary salt solution (NaCl) of arbitrary though relatively high concentration is injected into a stream, the water at a point downstream will be a dilute salt solution during a certain period after the injection. The dilution which the primary solution has undergone may be determined by measuring the variation of the conductivity of the water during the time the "salt wave" takes to pass the point of measurement.

The amount of the primary salt solution is adjusted according to the size of the stream and the natural salt content of the water. In favourable circumstances 50 litres of salt solution are sufficient to measure a discharge up to 50 m^3/sec.

For measuring the discharge of a river the brine is injected at a place where there is such strong turbulence that a homogeneous mixture can be obtained within as short a distance as possible. The injection need not be continuous for a certain period; it can take place instantaneously or intermittently during a short period of time.

Mixing with the running water produces salt solution with a variable degree of dilution. This dilution is measured continuously at the measuring point as long as the salt wave is passing.

The recording device is an instrument for measuring electrical conductivity, coupled to plate electrodes immersed in the water at the place of measurement. A suitable instrument is an a.c. soil-resistance meter (Megger earth tester). The size and shape of the electrodes is adapted to the salt content of the water. The electrodes should be of silver to reduce the effect of polarisation.

Resistance and time must be recorded throughout the period of measurement. If an automatic recording instrument is not available it may be necessary to film the meter reading if the wave of salt passes rapidly. The temperature of the water is taken before and after each series of measurements. The resistance graph that may be constructed with the aid of the measured values differs in appearance according to the flow conditions.

The resistance graph can be transformed to a dilution graph in the following manner. A small sample of the injected brine is set aside so that its resistance may be determined at various dilutions. A container of known volume is filled with river water, the electrodes are immersed in the water, and the resistance (R_B) and temperature are read off. By means of a pipette b ml of the primary solution are then added to the water in the container, producing a solution of dilution $f_b = b/1\,000\,B$, if B is the volume of the container in litres. The resistance and temperature are once more recorded, and the same procedure is afterwards repeated with further additions of the primary solution, until a calibration curve is obtained that covers the range of values from the actual measurements. The calibration curve shows the relation between resistance and dilution.

Direct graphical application of the calibration curve, which is approximately a hyperbola, is not sufficiently accurate. It is therefore customary to use the following formula:

$$f_b = K\left[\left(\frac{1}{R_b} + \frac{1}{R_b{}^2}\right) - \left(\frac{1}{R_B} + \frac{1}{R_B{}^2}\right)\right] \tag{60}$$

If the expression $\left(\frac{1}{R} + \frac{1}{R^2}\right)$ is replaced by $\alpha\, R$ this formula becomes

[1] The account given here is in the main an abridged form of that in AASTAD and SÖGNEN 1952.

$$f_b = K(\alpha R_b - \alpha R_B) \tag{61}$$

The graph of f_b vs. α is a straight line according to this equation. The experimental curve from some rivers is slightly curved, but in any case it is not difficult to transform the resistance graph to a dilution curve graphically.

The area under the dilution vs. time curve corresponds to the average dilution of the primary solution at the measuring point, multiplied by the time taken for the salt wave to pass the point. The discharge is obtained from the formula

$$q = \frac{V}{A} \tag{62}$$

where q is the discharge, A the area under the dilution-time curve, measured in appropriate units, and V the amount of brine injected.

Corrections must be made for temperature variations and, where necessary, for the difference between the resistance of the river water measured in the actual channel and that measured in the container used for calibration measurements.

Application of the relative salt-dilution method to measurements of the turbulent diffusion

When the relative salt-dilution method is used to measure the discharge of a river, it is necessary to choose the place for injection so that the brine mixes with the river water as rapidly as possible in order to obtain a reliable result.

But if the method is to be used to study turbulence, the purpose is to follow the spreading in detail from the injection of the brine. In this case the method is not restricted to stretches of the river where there is rapid, turbulent flow; it can also be used where the flow is relatively calm. However, it is important that the injection of brine does not affect the local state of turbulence to such a degree that the mixing process does not follow its normal course.

The factor that may affect the natural mixing process is the difference between the density of the injected brine and that of the river water. The density depends on the salt content, the temperature, and the sediment content. Since the local river water is used to prepare the brine, the differences in temperature and sediment content are generally so small that they can be neglected. The salt contents, on the other hand, differ widely.

The natural salt content in rivers exhibits large regional and temporal variations (ERIKSSON 1929). In Fyrisån, for instance, the salt content varies about an average value of c. 200 mg/l (HJULSTRÖM 1935, p. 430), while in Klarälven it is some 30—40 mg/l (ERIKSSON 1929, p. 39). When the natural salt content is 200 mg/l, a solution with a density 0.0003 g/cm³ higher has a concentration two times as high; for a salt concentration of 40 mg/l the figure is ten times.[1]

If 0.0003 g/cm³ is taken to be the least density difference that can affect the turbulent mixing (cf. p. 157), the above estimates indicate that considerable relative differences in salt concentration may occur without causing density stratification. Nevertheless, it cannot be expected that the strong concentrations usually employed for river discharge measurements (10—20 % solutions) will not to some extent damp the local turbulence on injection.

[1] The figures are approximate, since the chemical composition of the solute and the temperature of the water affect the results to some extent.

In order to investigate the influence of the salt concentration on the mixing process repeated measurements were carried out at various distances (10—200 m) below a fixed injection point in Klarälven. The concentration of the injected brine was varied. The brine was poured directly into the water from a boat. The time taken for this operation was 2—5 sec. At distances less than 30—40 m from the injection point the instrument readings were generally filmed. The main results of these tests may be summarised as follows.

1. At short distances from the point of injection the readings fluctuated greatly during the short time the salt wave took to pass, especially when high concentrations were used.

2. At large concentrations it was found that the injected salt solution sank up to a metre through the less dense river water during the first part of its movement downstream.

3. At greater distances from the injection point (> 50 m) there were no detectable differences in the spreading out of the salt solution for different primary concentrations.

These observations seem to indicate that the injected solution at first moves to some extent independently, and that the microturbulence at the boundary surface between the solution and the river water is damped somewhat by the density contrast. But bodies of brine are soon deformed and broken up by macroturbulent motion in the flowing water, and an effective mixing takes place. Since the volume of solution injected is always very small compared with the river's discharge, the salt concentration soon falls to such a low value that there is no longer any risk of density stratification and modification of the turbulent mixing. The state of turbulence in the river naturally determines the efficiency of mixing to a large extent.

The turbulent mixing process can therefore be studied by injecting salt solution, *provided the amount of salt solution injected is small in comparison with the volume of flow, and provided the measurements are carried out sufficiently far downstream from the point of injection.* Moreover, the injection should not be made at a place where the water is almost stationary.

The fact that concentrated solutions of such an inexpensive substance as ordinary cooking salt can be used in this method facilitates the practical performance of the measurements to a high degree. However, it is worth noting that for detailed studies of microturbulence the injected liquid must have the same density as the river water (cf. KALINSKE 1940, referred to in the present publication on p. 150).

Performance of the measurements

Measurements by means of the salt-dilution method have been carried out at several places on Klarälven, as well as in several other Swedish rivers, e.g. Mörrumsån, Ångermanälven, Öre älv, and Ume älv. The field equipment was adapted to the particular purpose in each case and to the local conditions, especially the water discharge and the pattern of flow, and has therefore varied considerably from case to case.

Injection of brine: For small-scale experiments the brine was generally prepared in a stainless steel container. The salt concentration was usually c. 10 %. The brine was either poured directly into the water, or pumped through a rubber tube with weights attached to keep the end at a fixed depth below the surface (fig. 24). For experiments on a larger scale the solution was prepared in one or more 200 litre containers, which were subsequently

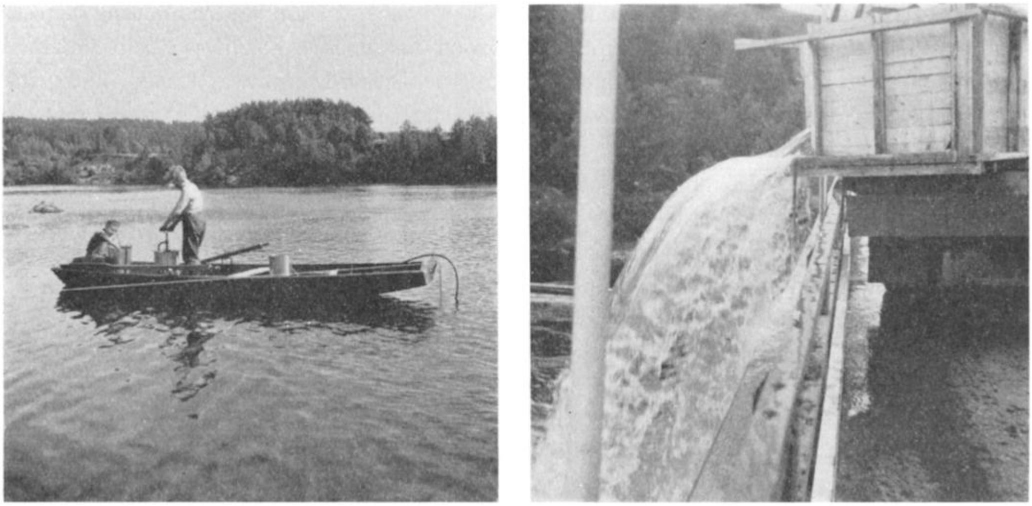

Fig. 24. Injection of brine. Left: brine being pumped through a rubber tube into the water at a fixed depth (Klarälven 1953). Right: 5,000 l. brine being poured into the river from the bridge at Forsmo (Ångermanälven 1954, photo L. ARNBORG).

emptied directly into the river. In two cases the amount of salt solution injected was as much as 5,000 litres 10 % brine. On one of these occasions a wooden container was mounted on a lorry, from which the brine was poured into the river from a bridge (fig. 24).

The measurements: The conductivity of the water during the passage of the salt wave was measured with the instrument shown in fig. 25. The electrodes have already been described. They were mounted on a 12 kg lead weight, which could be lowered to the desired depth. The electrodes were coupled to a soil-resistance meter.

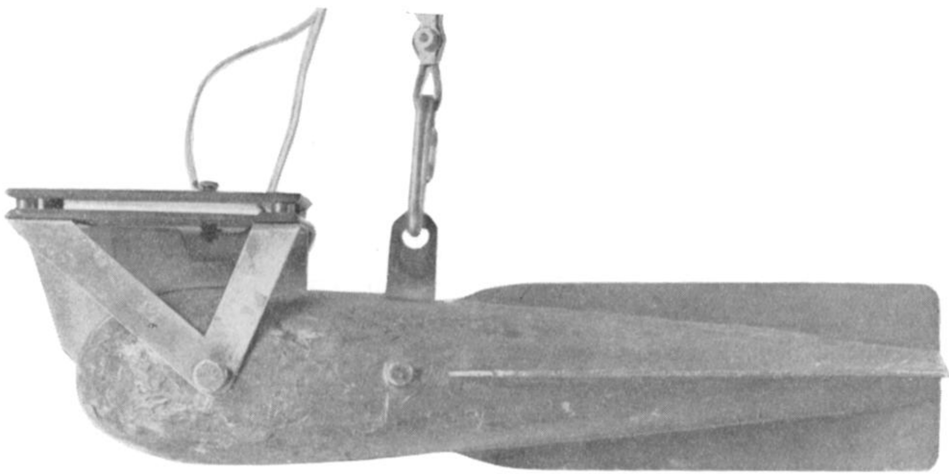

Fig. 25. The electrodes consist of silver plates with an area of c. 50 cm^2, embedded in 6 mm thick backelite plates and mounted on a 12 kg lead weight. The distance between the silver plates can be varied.

In order to investigate the turbulent diffusion in the river it was necessary to carry out measurements at a large number of points on transverse sections downstream from the point of injection. The best way to do this would have been to have simultaneous recording at a number of points, but the available equipment did not permit this. Instead, the same amount of brine was injected many times at the same point, while the measurements downstream were carried out at successive points. Subsidiary experiments were carried out to check the reproducibility of the spreading process.

When the distance between the injection point and the section for measurements was so large that the salt wave took more than 5 min to pass, the measuring apparatus was sometimes made mobile. The boat from which the measurements were made moved backwards and forwards along a marked line in the transverse section, and meanwhile time, position, and resistance were steadily noted. A similar method was used in some cases when measurements were carried out from a bridge.

Treatment of the data

A graph of concentration vs. time was constructed from the primary data for each point where measurements were taken. This graph differs from point to point, depending on how the salt solution had spread out by the time it reached the measurement section. From these separate graphs it was then possible to construct a graph of concentration vs. time for the entire section, showing the passage of the salt solution across the section (see figs. 26, 27, and 28).

From a knowledge of the concentration diagram at a particular point and the flow velocity there it is easy to calculate *the total amount of primary salt solution* passing through unit area perpendicular to the flow. The calculation of this quantity for each point where measurements were made gives the spread of salt solution in the transverse section. A diagram of the section with interpolated lines through points where the same amount of salt solution passed gives the probability that water particles passing through the point of injection subsequently passed through different parts of the section where measurements were made (see fig. 55, p. 283).

Some results from various localities

This section is concerned with some measurements and the data obtained from them, described with reference to a series of diagrams.

Fig. 26 shows the results of salt-dilution measurements in the stream Rutjebäcken near Rönäs on the lake Tängvattnet in Tärna parish (in the basin of Ume älv). Brine was injected at a point just downstream from the waterfall visible in the photograph, 5 m from the western bank of the stream. Measurements were made from the road bridge 147 m downstream from the point of injection. The stream is c. 25 m wide on this stretch, and the fall even, 0.82 m in all. The bed of the stream consists of boulders and stones, with the largest boulders projecting above the water surface at the time of the measurements.

The sub-figure on the upper right shows the graphs of concentration vs. time for the points A, B, C, D, and E. Each of them has the shape typical of such graphs: first a fairly

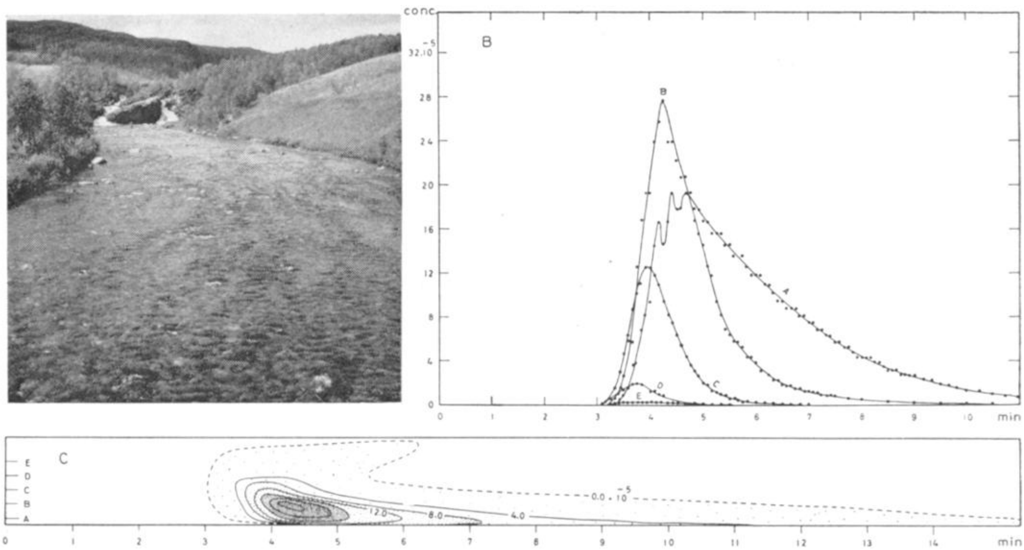

Fig. 26. Salt-dilution measurements in Rutjebäcken 1954. Upper left: photo from the stretch investigated. Upper right: graphs of concentration vs. time for the five measuring points. The figures for concentration indicate the concentration of the primary salt solution during the passage of the salt wave. Lower sub-figure: the passage of the salt wave past the measuring section.

rapid rise in the concentration when the salt wave reaches the measuring point, followed by a slower fall of the concentration finishing in a long "tail".

The passage of the salt wave is more clearly illustrated in the lower sub-figure. The positions of the measuring points are shown on the left. A is approximately 2 m from the left bank. The other points are at intervals of 4 m. It may be seen that the first trace of salt reaches the section at point C 3 min after injection. The average flow velocity for the salt solution was 82 cm/sec. The crest of the salt wave, i.e. the highest concentration of salt, reached the section after 4 min 15 sec (average velocity 58 cm/sec). The salt concentration remained quite high near the left bank still 10 min after injection. The average velocity of the corresponding salt solution was less than 25 cm/sec.

The most surprising feature of the spread diagram is the lateral narrowness of the concentration peak. In spite of the eddies in the rapidly moving stream, there was relatively little lateral smoothing-out of concentration differences. Near the right bank there were only negligible traces of salt solution. There is also a noticeable tendency for the salt wave to lag behind near the banks, on account of the lower flow velocity there; the salt wave has a pointed front in the central part of the stream.

Fig. 27 shows the pattern of flow through Sandslån's log-sorting works where Ångermanälven runs out into the sea-inlet Nylandsfjärden. The upper figure is a plan view of the sorting station, and the lower figure a map of the storage pond in detail, together with the spread diagram for the salt wave. This particular investigation was undertaken to determine the effect of the storage pond on flow conditions by making measurements for different

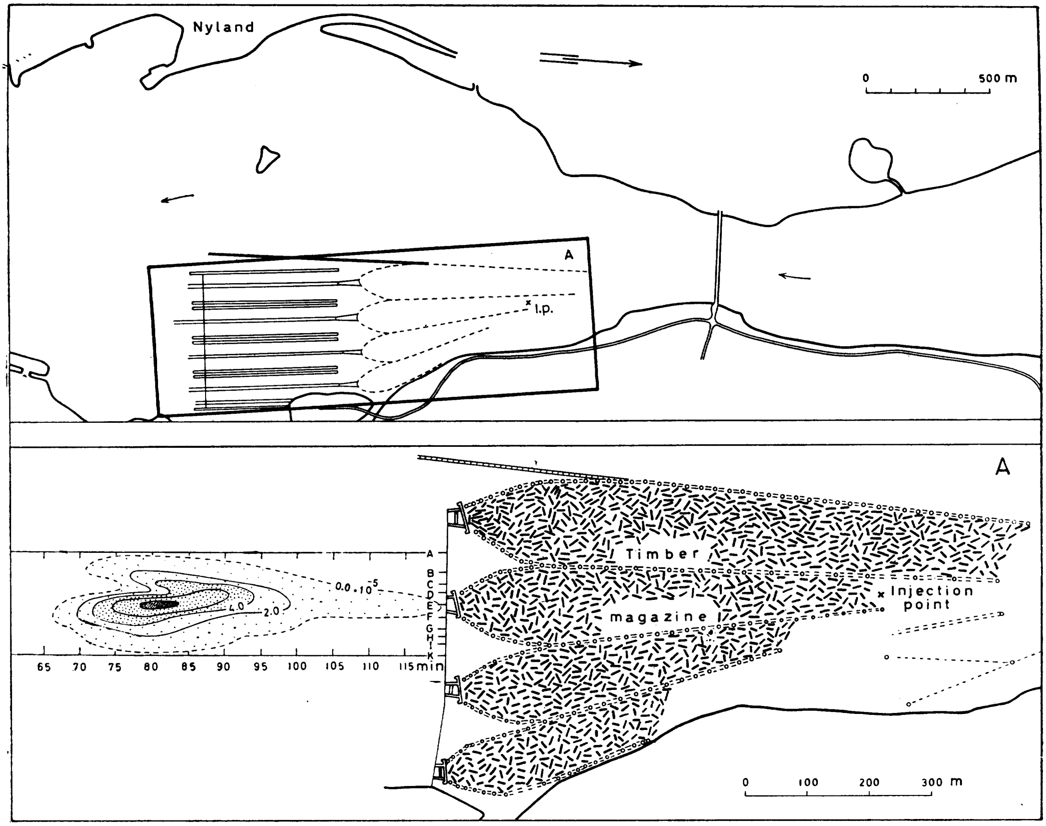

Fig. 27. Salt-dilution measurements in Ångermanälven at the log-sorting works at Sandslån. Further explanation in the text.

volumes of flow and different amounts of timber in the pond, and thereby to estimate the significance of timber for sedimentation in that part of the river.

Brine was injected immediately upstream from the storage pond (see fig. 27). Measurements were made 700 m from the injection point along a boom immediately downstream from the storage pond. The first traces of the salt solution were detected after 1 hr 6 min, corresponding to a flow velocity of 18 cm/sec. The highest concentration occurred after 1 hr 20 min (velocity 15 cm/sec), and the last traces of the salt solution disappeared after 2 hr (velocity 10 cm/sec). The lateral spread was 155 m, corresponding to an angle of spread of 12.6°.

There are small irregularities in the spread diagram, probably caused by rows of poles and the logs. Apart from these irregularities the diagram is more symmetric than in the preceding case. It is typical for a situation with two-dimensional flow, i.e. flow where edge effects due to the river shores may be neglected. In contrast to the former case the salt wave is most elongated near the centre. The spread diagram may be described as egg-shaped,

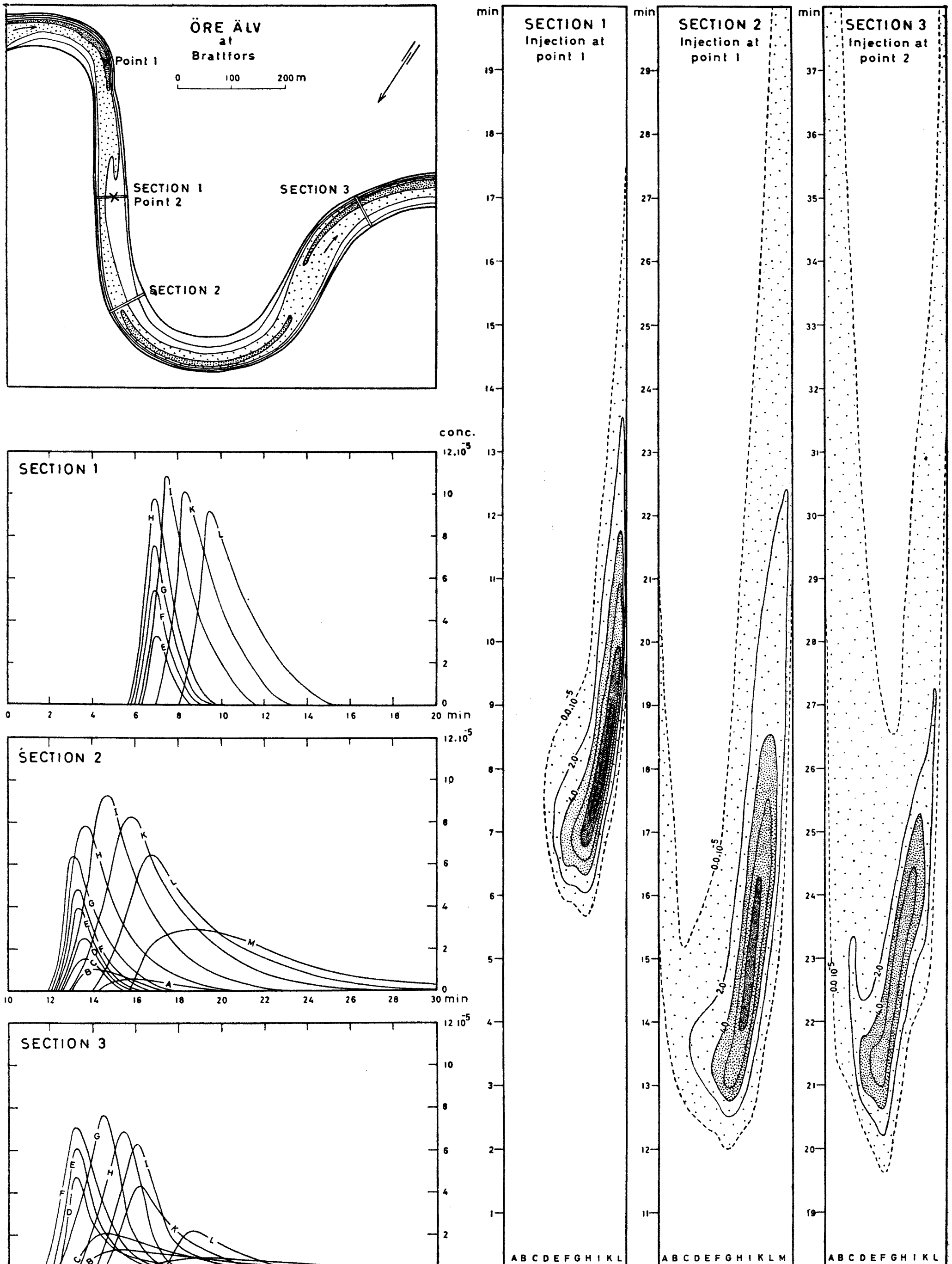

Fig. 28. Salt-dilution measurements in Öre älv. Upper left: plan view of the stretch investigated. Depth contours at intervals of 1 m, interpolated from B. MÖLLER's soundings. Further explanation in the text.

with the pointed end directed backwards. Moreover, the total spread in the direction of flow is less than in the preceding case, and is entirely due to the lower flow velocity near the bottom and near the floating timber.

The final example chosen to illustrate the application of the salt-dilution method is from an investigation in the meander zone of Öre älv near Brattfors. The positions of the injection points and of the sections where measurements were made is indicated in the sketch map on the upper left in fig. 28. The map also shows the river depth in the region investigated according to soundings by B. Möller.

The first part of this investigation involved injection at point 1 and measurement in sections 1 and 2. The graphs of concentration vs. time based on these measurements are shown in the two middle sub-figures on the left in the figure. The positions of the measuring points are shown at the bottom of the sub-figures on the right. The distance between the measuring points was 5 m. The left bank of the river (looking downstream) corresponds to the right side of each sub-figure.

The graph of concentration vs. time clearly shows the large variations in the passage of the salt wave past different points.The lateral mixing is imperfect. From the left-hand figure of the three on the right it may be seen that the first traces of salt solution reached section 1, 250 m downstream from the point of injection, after 5 min 40 sec (velocity 74 cm/sec). The highest concentration arrived after 6 min 40 sec (velocity 63 cm/sec), and even after 15 min there was a detectable concentration of salt near the left bank.

The initially small body of salt solution was thus stretched out so that it became asymmetric, with the point of the wave front on the main channel, where the flow velocity is highest, and the flanks lagging towards the shores. The width of the region over which the salt wave has spread in section 1 is 37 m.

In section 2, 460 m downstream from the point of injection, the salt solution has spread out over the whole section, but the region where the concentration is high is still quite restricted. The first trace of salt was recorded roughly in the middle of the section after 12 min (velocity 64 cm/sec), and the highest concentration arrived near the left bank after 14 min 30 sec (53 cm/sec). There were still traces of salt solution near the left bank after 30 min.

The pattern of spread now has an even more pronounced point in the centre of the main channel, with a time lag nearer the banks. The salt wave is still asymmetric, owing to the fact that the brine was injected near the left bank.

The other part of the investigation involved injection at point 2 and measurement in section 3 about 800 m downstream. In this case the river has flowed around a complete meander loop before reaching the measuring section. The first trace of salt was detected after 19 min 35 sec (velocity 68 cm/sec). The highest concentration arrived after 22 min 30 sec (59 cm/sec), and there were still traces of salt solution near both shores after more than 35 min. The spread of the salt solution clearly indicates how the flow varies in the meander loop. The lower flow velocity near the left bank appears as a lagging behind of the salt wave near this side. The river has already begun to turn in the other direction by the time it reaches section 3, and this is reflected in a lagging behind also along the right-hand side.

The general result of the measurements in Öre älv is that the small body of solution at the point of injection spreads out as it moves downstream to form a salt wave with a point along the main channel and flanks lagging behind towards the river shores. Consequently an increasing amount of the salt solution will spread out laterally away from the main channel towards the shores as the wave moves on. The salt content in the front of the wave gradually diminishes, until the salt wave ultimately fades out. This behaviour is important as regards the transportation and deposition of suspended material, and it provides possibilities of estimating how long suspended material of a particular grain size is likely to remain in suspension.

Some fields of application for the salt-dilution method

Measurements by the salt-dilution method provide a clear picture of the turbulent diffusion in a river. But as regards the quantitative evaluation of the results, it is a very complex theoretical problem to derive from the measurements the value of some parameter which may represent the intensity of the turbulent mixing. The theoretical problem is even more complicated for a river, where there are edge effects, than in the atmosphere, for instance. In this connection it may be called attention to the measurements of the spread of smoke and gas in the atmosphere that have led to valuable results concerning turbulent diffusion in the lower atmosphere (cf. SUTTON 1953).

But it may also be contended that a knowledge of the general manner in which the turbulent diffusion occurs is as important as, for instance, numerical values of the widely varying transfer coefficient for matter. Salt-dilution measurements show the velocity characteristics of the flowing water, and so of the suspended matter, and they indicate the lateral diffusion and the retardation of flow near the shores or in regions of calm water. Their significance for the study of sedimentation is thus evident. The method can also be used to determine how pollution spreads from outlets flowing into a river. In many cases it may be desirable to see whether water from one point in a river can reach another point downstream from the first.

Under certain conditions it is possible to study turbulent diffusion in a river by conductivity measurements without injecting brine, for instance, where two streams with different salt contents join. The mixing process may be observed by measuring the conductivity in transverse sections downstream from the junction, and the effects of variations in salt content, temperature, and difference in suspended load between the affluent streams can be studied.

The possibility of obtaining a quantitative estimate of sedimentation in a part of a river, lake, or reservoir is of course of particular interest. It was stated on p. 221 that the sedimentation on a relatively regular stretch of a river could be obtained from eq. (59) if certain simplifying assumptions were made. In order to be able to apply eq. (59) it is necessary to know the time t during which the water has been in motion along the stretch in question. The author has made use of a concentration vs. time graph constructed from salt-dilution measurements on the particular stretch of river in order to determine t. The numerical value of t is not fixed by this procedure; instead t is given in the form of a function that corresponds to the concentration vs. time curve. The computation of the sedimentation then becomes a matter of graphical integration, and the author hopes to have the opportunity of describing the actual technique in a future article.

Sampling suspended load

General

A good deal of attention has been paid to the construction of sampling apparatus for suspended sediment in flowing water. Among those who have discussed the principles for the construction of such samplers are JAKUSCHOFF 1932, and HJULSTRÖM 1939 b, Extensive work in this field was later carried out at the Hydraulic Laboratory of the Iowa Institute of Hydraulic Research in Iowa City in connection with a programme of research on the transportation of sediment by flowing water. This programme has included technical investigations of samplers for suspended load and for bed load, investigation of the procedure to be used for taking samples, and methods for analysing samples. The results of these investigations have been published in a series of reports (Measurement and analysis of sediment loads in streams, 1940—1948).

There are two main types of samplers: 1. *samplers filling instantaneously*, and 2. *integrating samplers.*

The concentration of sediment at a particular point in a river is not constant: it fluctuates widely on account of turbulence and the unevenness of the distribution of sediment in both the vertical and horizontal directions. Therefore a sample taken instantaneously does not give a representative value of the average concentration of sediment at the point in question. To obtain such a value from instantaneous samples it is necessary to have a whole series from which an average can be calculated. Such a procedure is time-consuming and expensive, and samplers filling instantaneously are therefore used as a rule only for special investigations where it is desired to study the fluctuations of the suspended load with time.

Alternatively, a representative value may be obtained by allowing the sampler to fill during a period sufficiently long to eliminate the effect of rapid fluctuations. The principles to be used in determining the time required for a representative sample have been worked out statistically by KALINSKE (1945).

Integrating samplers of several types have been constructed. Earlier an ordinary bottle was commonly used, sometimes surrounded by a container of heavy material; this bottle was lowered to the desired depth, and there opened in some way, after which it was allowed to fill and then drawn up. However, it was found that this type of sampler was liable to give erroneous results.

The modern types of integrating samplers may be divided into *point-integrating* and *depth-integrating*. In the former case the sampler remains at the same point while it is being filled, and the sample obtained may be regarded as providing a representative value of the sediment content at the particular point. In the latter case water flows into the sampler while it is being lowered at a steady rate from the surface down to the river bed and then up again, or alternatively only in the one direction. The sample thus obtained is taken to provide a representative value of the suspended load for the vertical line concerned.

An ideal integrating sampler should fulfil the following requirements (Measurement and analysis of sediment loads in streams, report no. 8, 1948, pp. 78—79).

1. "The velocity within the cutting circle of the intake should be equal to the stream velocity.

2. The intake should be pointed into the approaching flow and should protrude upstream from the zone of disturbance caused by the presence of the sampler.

3. The sampler should fill smoothly without sudden inrush or gulping.

4. The sample collected at a point should not be contaminated by water or sediment accumulated prior or subsequent to sampling.

5. The volume of the sample should be sufficient to satisfy the laboratory requirements for the determination of the concentration and size analysis.

6. The sampler should be adaptable for use in streams of any depth and for sampling at any desired depth.

7. The sampler should be streamlined and of sufficient weight to avoid excessive drag.

8. The sampler should be of simple design and yet sufficiently rugged to minimize the need for repairs in the field.

9. The sample container should be removable and suitable for transportation to the laboratory without less of any of the contents."

The simple bottle type of sampler is opened at the desired depth, whereupon the pressure difference causes water to rush in suddenly. At a depth of 10 m the bottle is half-filled within a second or two. Even if the bottle fills more steadily afterwards, it is obvious that the concentration of sediment at the instant of opening will dominate in the final sample. An arrangement which eliminates the pressure difference between the empty sampler and the surrounding water is therefore desirable.

Suspended matter follows the movements of the water bearing it on the whole, but not always. For the grains of sediment have a motion of their own corresponding to the settling velocity in the water, and they have a greater inertial resistance to accelerating forces than the water has, on account of their greater density.

The greater inertia of the sediment particles has an important consequence for samples. If the velocity of inflow into the sampler is less than the free flow velocity, the water will be diverted from the orifice to some extent. But the sediment particles do not follow the diversion exactly, which means that the concentration of sediment in the sample is higher than that in the stream. Conversely, if the inflow velocity is higher than the stream velocity, the concentration in the sample will be too low.

The error in the measured sediment content arising from this source varies with grain size, with the ratio of the inflow velocity to the stream velocity, and with the orientation and shape of the sampler's orifice. The graph in fig. 29 shows the percentage error in the concentration of sediment for various inflow conditions plotted against the grain size (after Measurement and analysis of sediment loads in streams, report no. 5, 1941, p. 40).

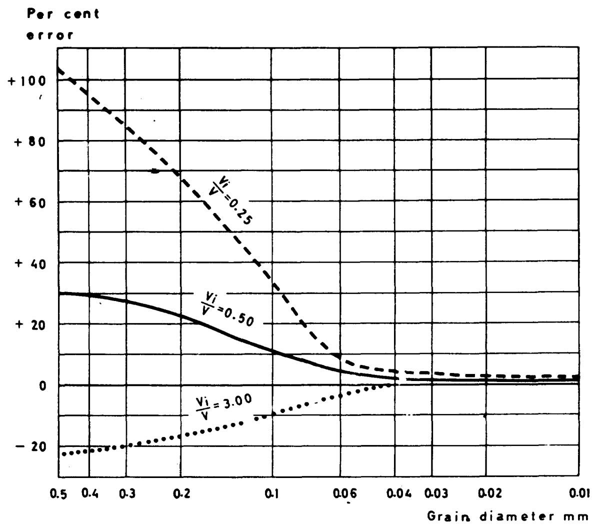

Fig. 29. Percentage error in the sediment concentration of water samples for various inflow conditions plotted against the grain size. Inflow tube pointing into the approaching flow. V_i = inflow velocity. V = flow velocity in the free water (after "Measurement and analysis of sediment loads in streams", Report No. 5).

The diagram clearly shows that for a grain size outside the region of validity for STOKES' law the percentage error is very large, especially when the inflow velocity is low. The stream velocity also affects the error to some extent. The error is smaller when the stream velocity is lower. The error increases somewhat for smaller orifices.

A sampler for suspended load

The author has designed and used a sampler which satisfies the essential requirements for such a device. The U.S. point-integrating suspended sampler, P-46, was unknown to the author when he designed this sampler in 1950. Although the two samplers are in principle similar, a description of the former is perhaps not out of place here.

A photograph of the sampler is shown in fig. 30. The outer envelope is a brass cylinder with a shutter giving access to the sampling bottle. At each end there is a lead weight, which, together with the brass cylinder, give a streamlined form. The total weight is slightly more than 20 kg. The sampling bottle is a half-litre round-bottomed flask, which can easily be inserted in or removed from the sampler.

The inflow tube passes straight into the flask through a two-way stop-cock. Another tube from the flask is connected by way of a three-way stop-cock with a rubber bladder and with an outlet tube. The two stop-cocks are gear-coupled, and can thereby be moved by means of a cord which goes up to the surface. A spring keeps the inflow and outflow tubes normally closed. The operation of the sampler is as follows.

The sample flask is first placed in position, and the rubber bladder is pumped up. When necessary its volume can be made 2 times that of the sampling flask. The spring now keeps both the inflow and outflow tubes closed, while the rubber bladder and the flask are connected.

The sampler is now lowered into the water with the help of a winch. As the hydrostatic pressure increases the bladder is compressed, and the air in the flask is thus maintained at the surrounding hydrostatic pressure.

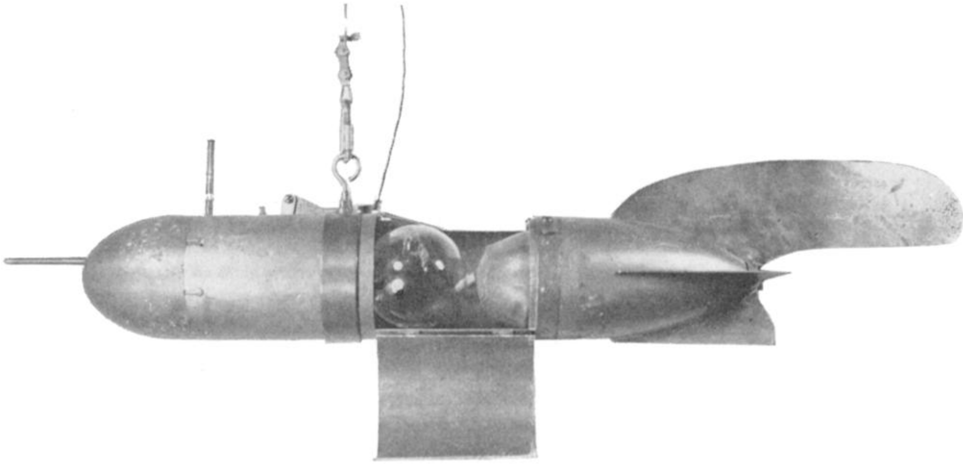

Fig. 30. The sampler for suspended load. Explanation in the text.

When the sampler has reached the desired depth the cord is pulled. The inflow tube is thereby opened, and at the same time the outlet tube is opened so that air can escape. Also, the connection between the bladder and the flask is broken. In this position the spring is stretched. When the sampling flask is full, further water is prevented from entering by a small float valve.

When it is certain that the flask has had time to fill, the cord is slackened, whereupon the inflow and outflow tubes are both closed. The sampler is hauled up, and the sampling flask taken out.

The inflow and outflow tubes are of such a form that the inflow velocity is very close to the stream velocity. The form was arrived and checked by flume experiments at the Hydraulic Laboratory of the Royal Institute of Technology, Stockholm. It was found that the velocities were the same within 20 % for flow velocities between 30 and 150 cm/sec, while the inflow velocity was somewhat too high at lower stream velocities and somewhat too low at higher.

In Klarälven the sampler was used for special measurements where high accuracy was required. It was not possible to use it for routine sampling at different sites on the meandering part of the river (p. 296), since only one sampler has so far been constructed. The routine samples were taken with an ordinary sampler of the bottle type, made by the Swedish Institute for Meteorology and Hydrology. Comparison of samples taken with the two types of sampler showed that the latter type gives a somewhat low value of the sediment load, but that the discrepancy is generally less than 10 %, and only in exceptional cases more than 30 %. In view of the actual small sediment contents and random fluctuations this difference cannot be regarded as serious.

Under-water photographs

In certain cases it has been found necessary to obtain more detailed information about conditions on the river bed than can be obtained by bed samples. This applies in particular to the study of minor morphological formations such as ripples and, to some extent, transverse bars, as well as to the arrangement of particles in the surface of the bed.

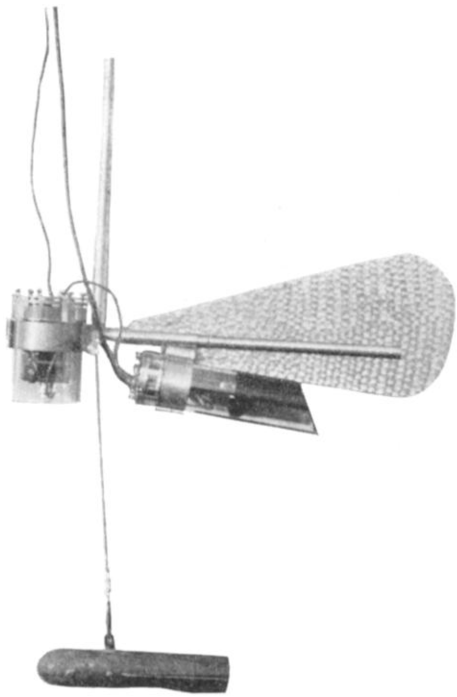

Fig. 31. Apparatus for under-water photography.
The length of the fin is c. 50 cm.

In order to be able to study the river bed at least in relatively clear water and when there is moderate transportation of sediment, the author has constructed an apparatus for under-water photography. A photograph of the apparatus is shown in fig. 31. The camera is a Bell and Howell 16 mm film camera. It was set to take single pictures, and equipped with remote control of the shutter. It was synchronised with an electronic flash. A timer can be coupled to the shutter to give exposures at required intervals. Both the camera and the electronic flash lamp are enclosed in boxes made of perspex as shown in the photograph. The apparatus also has a fin and a weight to give stability in the flowing water.

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CHAPTER IV

A STUDY OF THE FLUVIAL MORPHOLOGY OF KLARÄLVEN

INTRODUCTION

Klarälven is one of the most thoroughly investigated of the larger Swedish rivers, both geographically and geologically. An exceptionally comprehensive account of the morphology and stratigraphy of the upper valley is to be found in the paper "Klarälfvens serpentinlopp och flodplan", by STEN DE GEER (1911). Some earlier papers, in particular those of TÖRNEBOHM 1884, HOLLENDER 1900, DAHL 1902, and DE GEER 1906 b, deal with various aspects of the valley's geology.

In connection with the mapping work of the Swedish Geological Survey for the geological maps (series Aa), a thorough study was devoted to the southernmost part of the river, including the delta on the lake Vänern (MAGNUSSON and SANDEGREN 1933, SANDEGREN and MAGNUSSON 1937). In a further review article SANDEGREN (1939) has dealt with the post-glacial history of the river's lower valley.

During the 1940's VON POST planned an extensive investigation of the valley of Klarälven, with particular attention to the terraces in the upper part. In a preliminary communication (1948) he described the main features of the planned work, together with some preliminary results. But owing to the decease of VON POST in 1951 the project was shelved.

GILLBERG (1952) has studied the highest coastline in the valley of Klarälven as a link in his investigation of the process of deglaciation in western Sweden.

During the past few years, a geological map for the county of Värmland has been under preparation by the Swedish Geological Survey. The work, which has been led by J. LUNDQVIST, is now almost complete, and the map with accompanying description should be printed during 1957. It is obvious that the geology and morphology of Klarälven will be treated at considerable length in the description.

The work carried out by the author in the valley of Klarälven was started in 1952. Since 1953 it has been carried out in collaboration with the Swedish Geological Survey. Primarily it was directed to a study of fluvial processes. The aim was to attain a better knowledge of fluvial morphology and stratigraphy. Two preliminary reports from this work have already been published (SUNDBORG 1954 b and 1956 a).

In view of the relatively extensive literature on the valley of Klarälven, as well as the geological survey now nearing completion, this descriptive chapter will not be an exhaustive account of every morphological and stratigraphical observation in the region under consideration. After an introductory description of position, climate, hydrology, and general geology, the remainder of the chapter deals principally with fluvial processes and morphological and stratigraphical problems intimately connected with these processes.

MAIN GEOLOGICAL FEATURES

Drainage basin

Klarälven is the most southerly of the larger Swedish rivers. It rises in the mountains of Härjedalen, passes through the lake Rogen, crosses the border into Norway, where it flows south through the lakes Fæmunden and Isteren, after which it again enters Sweden

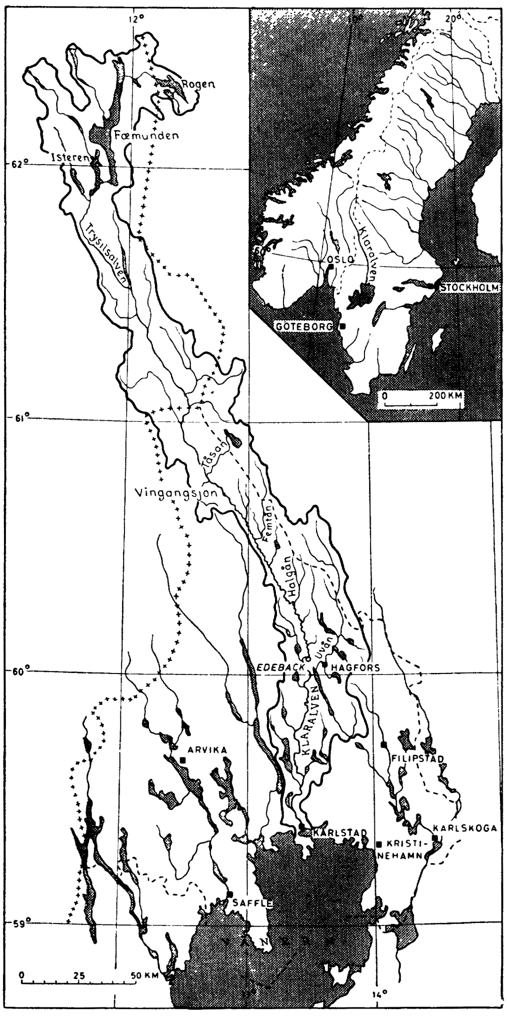

Fig. 32. The drainage basin of the river Klarälven.

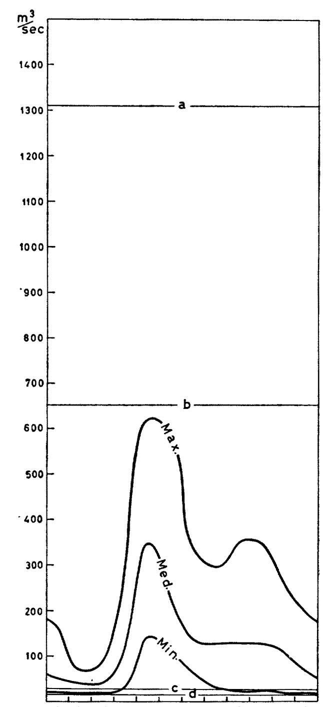

Fig. 33. Run-off characteristics at Edebäck. Max.: highest monthly mean values recorded. Med.: normal monthly mean values. Min.: Lowest monthly mean values recorded. a: highest discharge recorded. b: normal high-water discharge. c: normal low-water discharge. d: lowest low-water discharge recorded.

in the north of Värmland. Its course in Värmland is south-east or south down to the river mouth on Vänern at Karlstad (fig. 32).

At the Norwegian-Swedish border the drainage area comprises 5,420 km², and at Karlstad 11,820 km² (Förteckning över Sveriges vattenfall 108, 1932, p. 2). The largest tributaries in Värmland are Varån, Tåsan, Halgån, and Uvån. Lakes constitute 7.9 % of the drainage area above the Norwegian border, at Edebäck 5.9 %, and at the river mouth at Karlstad 6.6 % (De svenska vattendragens arealförhållanden 13, 1950, pp. 16—17).

The bed rock

The bed rock in the Värmland part of the river basin is predominantly granites and gneisses of Gothian age.[1]

East of a N—S line from the north of Värmland, running to the west of Klarälven down to the neighbourhood of Edebäck and then on south to Kristinehamn, are the Filipstad granites. This rock varies from basic types containing much plagioclase and dark minerals, primarily biotite and hornblende, to acid, reddish types containing much quartz and microcline but little dark minerals. Accessory minerals are magnetite, titanite, and apatite.

To the west of the line mentioned above are the gneisses of south-western Sweden, which, like the granites, vary in type from basic to acid, with mineral compositions similar to the granites. A characteristic of these gneisses is that they contain phacolitic or laccolitic masses of hyperite, lying parallel to the adjacent gneiss structure. Mineralogically the hyperites are principally made up of plagioclase, olivine, and pyroxenes. Other fairly common minerals are magnetite, ilmenite, biotite, and apatite (MAGNUSSON 1937, p. 28).

The black sands occurring in recent fluvial and lacustrine sediments, mentioned previously on p. 199, arise by mechanical enrichment of accessory minerals from the bed rock.

The tectonic valley. The diversion at Edebäck

Between the lake Vingängsjön and Edebäck Klarälven follows an almost straight course in what has been interpreted as a tectonic valley (cf. DE GEER 1911, p. 17, and MAGNUSSON 1949, p. 83). This valley does not coincide with the boundary between the easterly granites and the westerly gneisses, this latter being regarded as a limit of overthrusts which is now not visible superficially. The tectonic valley continues from Edebäck towards the south-east, down to Kristinehamn. In this part of it occur the lakes Rådasjön, Lidsjön, and Grässjön.

It is possible that in preglacial times the river followed the tectonic valley throughout its length (HOLLENDER 1900, DAHL 1902, LUKSEP 1954). But filling of the valley by the deep glaciofluvial deposits at Brattforsheden (HÖRNER 1927) hindered flow in the old

[1] TÖRNEBOHM mapped most of this region in 1880. Modern surveys of the bed rock are that of MAGNUSSON, for the sheets titled Filipstad, Nyed, Karlstad, and Forshaga of the Aa series of geological maps (1928, 1929, 1933, and 1937), and that of LJUNGGREN (1954) for a minor region on the border of Dalarna and Värmland. MAGNUSSON (1949 and 1954) has given general descriptions of the geology of Värmland.

preglacial channel, and a breach of the deposits in the neighbourhood of Edebäck during the early postglacial period (LUKSEP 1954) diverted the river through a gap in the hills at Edsforsen, whereafter it created its present course down to the delta on Vänern at Karlstad. However, the question of the supposed preglacial channel cannot yet be regarded as fully answered.

Postglacial development

At the time when the last Scandinavian ice-sheet was disappearing from northern Värmland, some 9,000 years ago during the late Yoldia period and the Ancylus period, a narrow inlet extended from the sea to Norra Finnskoga in the upper valley of Klarälven, 30—40 km from the Norwegian border. The most northerly limit of the highest coastline was at Båtstad in Norra Finnskoga, 220 m above the present level, according to GILLBERG (1952, p. 95). Southward the highest coastline falls at first fairly steadily, at Ekshärad being 180 m above sea level.

In the narrow fjord which then stretched northward from the Edebäck region, huge quantities of sand and silt were deposited. As the land continued to rise these deposits rose above the water in the fjord, and the former fjord gradually became a valley with a river running along the bottom of it. The river cut a deepening channel in the loose deposits, giving rise to the natural terraces of the present valley (DE GEER 1911, p. 17). VON POST (1948, pp. 197—8) considered that the terraces could be regarded as marine shore terraces, but this opinion does not seem to be borne out by published observations. The question will be discussed in a later section (p. 304).

The greater part of the eroded sediment was carried south and deposited as deltas on the gradually receding shores of Vänern, at Norra Råda, Munkfors, Övre och Nedre Ullerud, Grava, and Karlstad. The river channel has cut down into these delta deposits, with a continual redeposition of sediment in consequence.

In several places the river has reached underlying moraine or bed rock, whereupon its erosive activity in these places diminished or stopped entirely. But in other places the river still flows over easily eroded deposits. This is especially true of the stretch between Vingängsjön and Edebäck. It is this section of Klarälven that is the main subject of the present investigation.

CLIMATE AND HYDROLOGY

Climate

The climate changes considerably from the south to the north of the river basin. The region near Vänern is easily accessible to maritime air masses, while to reach the northerly part of the basin westerly winds have to pass over the mountains and plateaus of southern Norway. The climate therefore approaches a continental type as one moves northward.

The most important climatic features from the hydrological point of view are the annual variation of precipitation and of temperature. The temperature largely determines the rate of evaporation, thereby affecting the runoff, and it determines the permanency of snow cover. The accumulation of precipitation as snow decides the volume of the spring flood.

The annual precipitation in the river basin varies from 500—600 mm farthest south and in the area around the Norwegian lakes Fæmunden and Isteren, which are partially protected from rain and snow by surrounding mountains, to 800—900 mm in some parts of northern Värmland (Förteckning över Sveriges vattenfall 108, 1932, JOHNSSON 1937 and WALLÉN 1951). The late summer has the highest precipitation. Local variations depending on topography are large, however, and it is probable that the extreme values of the precipitation lie far outside the values given above. However, the network of stations recording precipitation is not close enough to give a proper indication of the local variations.

The annual mean temperature falls continuously from about + 6° C in the south to about — 1° C in the north of the drainage area (ÅNGSTRÖM 1938 and 1946). The table below gives examples of the annual variation in precipitation and temperature. The stations are arranged in order of position from south to north. Figures for months with an average temperature less than ± 0° C are italicised.

Table 1.[1]

Station		J	F	M	A	M	J	J	A	S	O	N	D	Year
Karlstad	T	*— 3.2*	*— 3.1*	*— 0.5*	+ 3.9	+ 9.9	+ 14.6	+ 17.3	+ 15.2	+ 11.1	+ 6.2	+ 1.4	*— 1.9*	+ 5.9
	P	*42*	*30*	*36*	39	50	51	71	88	56	63	53	*52*	631
Ekshärad	T	*— 7.1*	*— 5.7*	*— 2.3*	+ 2.3	+ 8.0	+ 12.4	+ 15.5	+ 13.3	+ 9.0	+ 3.7	*— 1.8*	*— 5.3*	+ 3.5
	P	*43*	*29*	*36*	33	45	55	70	99	57	52	*52*	*48*	619
Likenäs	T	*— 8.1*	*— 6.9*	*— 2.8*	+ 1.8	+ 7.9	+ 12.8	+ 15.0	+ 12.8	+ 8.6	+ 3.5	*— 2.6*	*— 6.8*	+ 2.9
	P	*48*	*32*	*34*	36	52	60	92	110	66	72	*58*	*50*	710
Storsätern	T	*— 10.4*	*— 9.1*	*— 5.5*	*— 0.8*	+ 4.3	+ 9.3	+ 12.1	+ 9.7	+ 5.5	+ 0.0	*— 6.0*	*— 9.2*	± 0.0
	P	*35*	*23*	*26*	*24*	46	56	79	92	52	51	*42*	*42*	568

The duration of the snow cover increases with decreasing temperature from about 90 days near Vänern to about 200 days in the northernmost part of the river basin. Its maximum depth also increases from south to north, from about 30 cm to about 90—100 cm (Atlas över Sverige 31, 1953).

Hydrology

The seasonal variation of the discharge of the river is associated with conditions of precipitation and temperature, especially the accumulation of snow during the winter. Since the thaw period is relatively short over the greater part of the drainage system the spring flood is violent but comparatively brief. The absence of any large storage volume in the form of lakes downstream Fæmunden and Isteren contributes of course to this behaviour.

At Edebäck the culmination of the spring flood occurs on an average 37 days after its

[1] Mean values for the period 1901—30 according to ÅNGSTRÖM (1938) and WALLÉN (1951). Storsätern lies in the drainage basin of Dalälven, but is only 30 km from Rogen, the lake where Klarälven rises.
T = monthly mean temperature in ° C, P = monthly precipitation in mm. The meteorological station in Karlstad is 53 m above sea level, Ekshärad 179 m, Likenäs 160 m, and Storsäter 680 m.

commencement, and at Forshaga about 20 km north of Karlstad 42 days (BERGSTEN 1940). The spring flood usually comes in the middle of May, but in exceptional years at the end of April or at the beginning of June. Once the spring flood is over, the volume of flow is mainly determined by the rainfall in the various parts of the drainage system. It may vary within wide limits, and on rare occasions even attain the same level as during the spring flood.

Fig. 33 shows curves of the volume of flow at Edebäck during the period 1889—1949 (according to the data and computations of Uddeholms AB). The upper curve represents the highest monthly values recorded, the middle curve the normal average values, and the lower curve the lowest monthly values recorded. It is obvious that all details of the annual variation disappear when average values are taken. The highest high-water value, the normal high-water value, the normal low-water value, and the lowest low-water value are therefore inserted in the figure.

The variations in the river stage which are associated with changes in the discharge are also of importance for morphological processes. The table below gives some representative values of the water-level at four places in the meandering course between Vingängsjön and Edebäck. The figures are derived from information in a project for damming at Edsforsen, and refer to natural conditions without damming (Vattenbyggnadsbyrån 1947). Since the river was dammed at Edsforsen the water level is now kept at 135.50 m above sea level at the river gage at Edebäck during the winter, and during the summer at 135.00 m.

Table 2.

Discharge (m^3 sec^{-1})	River stage			
	Fastnäs	V. Tönnet	Sälje	Edebäck
50	138.2	136.9	135.1	133.8
150	139.1	137.7	135.8	134.6
600	141.6	139.9	138.0	136.6
1,320	144.6	142.8	140.5	138.8
Difference (1,320—50) .	6.4	5.9	5.4	5.0

The most northerly locality, Fastnäs, is 26 km north of Edebäck. Since the entire meandering course is 80 km long, without rapids or falls, the variation of the water-level is much larger in the upper part. It has not been possible to obtain exact figures, but scattered data from the large spring floods of 1916 and 1931 indicate an amplitude of variation between 7 and 8 m farthest north (BÄCKVALL's Elfdals-arkiv, vol. 26, and FALK 1953).

The above figures for the discharge and water-level refer to undisturbed flow without river-control works. Power stations that have already been built, have altered the discharge a great deal, and projected stations will do so to an even greater extent. The general tendency will be that the maximum values are lowered, and the minimum values raised, that is, the seasonal variations of the discharge will be smoothed out to some extent. The implications of this for morphological processes is discussed on p. 309.

THE REGION INVESTIGATED

As already mentioned, the field work for the present study was mainly carried out in the meander course of Klarälven between Vingängsjön and Edebäck, the region previously described by De Geer (1911).

The present river channel has cut deeply into the sediment of the ancient fjord. The difference in height between the highest terrace and the river level is greatest in the north, at Vingängsjön, where it is about 54 m (De Geer 1911, p. 20). At Ekshärad, in the south, the highest terrace is about 35—40 m above the surface of the river. Thus the highest terrace slopes more steeply than does the surface of the river. This is partly due to the uneven uplift of the land. The present channel falls only 10 m over the entire meander

Fig. 34. Between Vingängsjön and Edebäck the river has developed an exceptionally regular meander pattern. The valley is almost exactly straight, except in a few places. The photograph shows the irregular course at Fastnäs-Åstrand in Norra Ny parish. In the foreground an abandoned meander loop. Photo Försvarsstaben.

course, a distance of some 100 km if one follows the winding river. The river flows steadily without rapids, with an average gradient of 10 cm per km.

The average width of the valley bottom is 1 km (cf. Pl. I), and the mountain sides bounding the valley are almost exactly straight. The course of the river is an exceptionally regular sequence of meanders, unique in Scandinavia. The meander loops divide the valley floor into a series of relatively low-lying headlands.[1] These headlands provide most of the agricultural land of the region, forming at the same time natural units of the local settlement and are the main sites of the river's morphological activity (fig. 34).

There are many examples of meandering rivers and streams in Sweden (cf. the map in HJULSTRÖM 1942), but the meandering part of Klarälven is unique in its size and regularity. Another unusual characteristic is the manner in which the narrow valley prevents the free development of the meander loops. Where the river approaches the valley side it is compelled to bend abruptly, often almost at right angles.

However, the river channel is not stationary, and a continual redeposition of the valley's sediments is in progress. The river removes material from the outside of each bend, i.e. from the north side of each headland. The eroded material is transported downstream, either in suspension or as bed load and is deposited in some place where the flow conditions favour deposition. Hence the north side of each headland is a steep erosion scarp while the south side is a shelving slope, on which there are alluvial ridges[2] parallel to the shore.

The processes of erosion and sedimentation cause each meander loop to shift gradually downstream, accompanied by the shore line of each headland. The whole system of meanders drifts very slowly down the valley. The land surface acquires an undulated form as new point bars are laid down. The headlands are everywhere of this undulatory form, which indicates that they have been subject to the reshaping activity of the river throughout their breadth (see Pl. II).

It is likely that the people living in the district had arrived at this view of the headlands' history long before anyone studied the river systematically. MÖRNER wrote in his description of Värmland (1762): "Autumn or winter rye is not grown. It is said that it cannot be raised near the river, or not with any prospect of success, since all the cultivable land is formed by the river flowing there, or removed and deposited elsewhere by the changeable river, so that it everywhere consists of small hills and dales following one upon the other, which give an appearance of waves on the sea during a violent gale."[3]

The excellent opportunities for studying the stratigraphic structure of alluvial sediment in the erosion scarps of the various headlands, where the present river cuts through deposits that it laid down during earlier phases, as well as the regularity of the meander pattern and its development, mean that Klarälven provides very good localities for examining fluvial processes and morphology. Another feature which makes the region a

[1] The swedish term is *näs*. Cf. ness, naze. There does not exist any exact synonym in English for the Swedish *näs*. The *näs* is here a lobe projecting from the valley side, and bounded on the other side by a meander loop (cf. Pl. I). In the sequel the English term *headland* has been used for *näs*.

[2] DE GEER called these alluvial ridges *älvvallar*. In the sequel the American term *point bars* will be used.

[3] Quotations from Swedish texts are translated to English throughout this book. Most of these quotations are to be found in Swedish in SUNDBORG 1956 a.

suitable site for field studies is that it has already been described in considerable detail, so that it is possible to make comparisons with the situation almost 50 years ago.

PROBLEMS

The field work was focused on certain particular problems which were judged to be of especial significance for an understanding of fluvial processes. Many individual observations have been made outside these main problems, and they are reported at various places in the general text.

The main problems may be briefly stated as follows.

1. How fast has erosion proceeded in the meander zone in geologically recent time? Is the rate of erosion roughly the same for all the meander loops, or are there large local variations? In the latter case, what factors favour erosion, and what factors retard it?

2. Is it possible to deduce the main features of the morphological processes by examining the morphology of the river bed and the meander headlands?

3. What conclusions regarding the formation of the meander headlands may be drawn from the stratigraphy?

4. Is it possible to obtain a detailed picture of the interrelation between river flow and the processes of erosion, transport, and deposition? What are the morphological and stratigraphical consequences?

5. How extensive is the present transportation of suspended material and bed load?

6. Is it possible to forecast the future geological evolution of the river and the valley?

7. Does a knowledge of what is happening at present help towards a clarification of earlier phases in the evolution of the river?

8. Do the investigations enable us to state more precisely problems worthy of special attention in the future in order to achieve a more reliable analysis of fluvial morphology and fluvial stratigraphy?

CHANGES IN THE SHORE LINES OF THE RIVER SINCE C. 1800

Sources of information

The question of the morphological effect of erosion and sedimentation during recent time requires a detailed knowledge of how erosion has progressed in various parts of the meander zone. There can be few other rivers than Klarälven for which the changes in position can be followed in such detail during recent centuries with the aid of old records and maps.

Records from the seventeenth century contain information indicating that some headlands or hamlets had suffered severely from the river's inroads[1]. The Jordebok (tax record) of 1639 says of the hamlet Spickebol in Norra Ny, that "it is devastated, and was deserted several years ago, since the river had cut away and destroyed the land".[2] People frequently complained that crops had failed on account of "frost and cold and the river flooding over its banks" (Allmogens ensk. besvär 1644, Riksarkivet).[2] "In the Dombok (court rolls) of Älvdal härad for the 3rd March 1636 it is reported that the people dwelling near the river in Norra Transtrand and Södra Transtrand, and in Slättne and Höljes in the then parish of Dalby complained that the river had cut away a good part of their arable land and pasture, for which reason they petitioned the magistrates and twelve sworn men to recommend an adjustment of their taxes to 'the authorities'. On the 20th March 1651 the härad magistrates reported that the river had carried away so much arable and pasture land from Öjenäs in the parish of Ekshärad, that it no longer sufficed for half a freeholding".[2]

However, it is only with the help of old maps that the changes in the course of the river can be traced in any detail. The oldest maps that are of use are those of J. P:son Thorsing, from the middle of the seventeenth century: "Delineation över gränsen mot Norge med angränsande socknar i Värmland" 1654—6?, and "Geografisk karta över Frisdalen och Älvdalen" 1658 (in the archives of Lantmäteristyrelsen).

These maps provide a general picture of the course of the upper Klarälven. The various meander loops and headlands can be distinguished, and their general appearance is indicated. One interesting observation is that the meander loop at Stärnäs-Lillängen in Dalby (see Pl. I) appears to have been already cut off at that time. The headland Björby in Norra Ny parish appears on the map to have been much larger than it is now. Habitation was confined to the northernmost part of the headland. It seems to have been a kind of double headland, more or less like Värnäs today. This indicates that Björby has undergone rapid erosion since then, leading to the disappearance of former farms and at the same time extension of the southern part of the neighbouring meander headland, Värnäs. However, it is not possible to estimate the area of the eroded land from these maps.

There are no geometric maps of the upper valley from the seventeenth century, apart from some geometric surveys of arable land and meadows in the hamlets of Norra Ny parish from the year 1697. These maps were made by the surveyor E. Wallringh. In the main they are accurately drawn, with arable strips and ditches clearly marked (cf. Nilsson 1950, p. 32). But the errors in the contours of the headlands are so large, even where arable land borders the shore line that it has not been possible to relate the maps to other maps of the same villages in attempting to investigate the movements of the shore lines. The same applies to later surveyors' maps from Dalby, Norra Ny, and Ekshärad, made by L. Gillberg, Ch. Roman, and C. F. von Hiltebrandt during the period 1719—1730. Neither have old county and provincial maps to be found in the collections of Krigsarkivet proved to be of any use for comparative purposes.

[1] In Lars Bäckvall's Elfdals-arkiv, vol. 26, Nordiska Museet, there are some hundred copies of old records and documents (jordeböcker, domböcker, allmogens ensk. besvär, landshövdingeberättelser, and other sources), referring to the damage done by Klarälven during the 17th and 18th centuries.
[2] From Nilsson 1950 (cf. also Bäckvall's archive).

The first maps to achieve an accuracy that permits detailed comparison with present-day maps are those made in connection with the *Storskifte*[1]. Practically all arable land and meadows in this part of Värmland was reallocated during the period 1760—1827, and more than half of the transactions took place in the period 1796—1805. During the *Laga skifte* later in the nineteenth century almost all the villages were surveyed once more, at least as accurately as for the Storskifte. The Laga skifte took place during 1828—1865, with occasional later redistributions. The majority of the transactions occurred between 1841 and 1855.

Since the middle of the nineteenth century a great many maps have been drawn up in connection with division of farms, change of ownership, and other transactions requiring surveys. Most of these maps cover only a small part of a headland, however. Although they might provide exact information about changes in a small region, they could hardly be used to compose a complete picture of this whole section of Klarälven at a definite time. Accordingly, they have been used only in exceptional cases, to supplement information from the maps of the Storskifte and Laga skifte.

The river from Vingängsjön to Edebäck has recently been photographed from the air (from Vingängsjön to Värnäs 1942, from Stackerud to Edebäck 1950, and the intervening stretch 1954), so it is possible to compare the older maps and these aerial photographs in order to trace changes in the river's shore lines during relatively well-defined periods. The year in which various areas were surveyed may differ considerably from one meander loop to another, but the general picture is not disturbed by such time differences. The result of the comparison of the different maps is shown in the map of Pl. I.[2]

Contents of the map

Land eroded during the period under consideration (c. 1800—1950) is coloured red. Blue denotes new land resulting from deposition of sediment. Dotted lines denote the river shores at the time of the Storskifte (c. 1800), dashed lines the shores at the time of the Laga skifte (c. 1850), and continuous lines the shores according to the aerial photographs (c. 1950).

River stretches where the position of the shore line has remained unchanged in the time between two consecutive surveys have in all cases been denoted by the latest contour. A continuous line by itself therefore means that no noticeable erosion or deposition has been traced during the period under consideration.

The year for the reallotment of land is given on each headland. When the records do not mention the actual date of the survey, the date given is always that for the commencement of the transfer of land. It is certainly exceptional if the date given on the map differs by more than one or two years from the actual year the land was surveyed.

[1] *Storskifte* refers to a reallotment of land whereby the land belonging to each owner in a hamlet was gathered together in larger fields, instead of the strips of the common-field system. The *Laga skifte* reform went further, and aimed at gathering together the land in each holding around the farm buildings.

[2] Similar comparisons of old maps in order to study changes in shore lines have been made by DE GEER (1906 a) for the erosion by the river Dalälven downstream from Älvkarleby, and by DE GEER (1911) for the headland Ljusnästorp in the valley of Klarälven. ENEQUIST (1944) has studied the migration of dunes in the Lule archipelago.

Places where active erosion is at present in progress are marked by spiked lines along the present shore line. It will be seen from the map that in almost all cases these sites are places where the shore line has previously been moved a considerable amount.

The valley bottom with its low meander headlands is in a light grey tone. The continual changes in the course of the river have in several places led to the cutting-off of meander loops. These loops have subsequently been filled with flood sediment and organogenous material. The position of these former meander loops is indicated by the symbol for marshland and by the remnants of ox-bow lakes here and there, especially in the southernmost section of the map (in the parish of Ekshärad).

The remnants of the old sedimentary deposits in the valley are indicated by darker shades of grey. These remnants lie along the sides of the valley, where they mostly form fairly narrow terraces. The darkest shade of grey indicates the highest terrace level, mostly representing the last remnant of the bottom of the ancient fjord. The slightly lighter grey denotes the terraces left by the successive downcutting of the river channel into the fjord deposits. It should be noted that the two colours for terraces do not imply that terrace levels in different parts of the valley have been correlated. They are merely a conventional manner of showing the main features of the morphology of the valley.

The terraces follow in the main the geological maps which are part of the results of the soil survey of Värmland at present in progress. These maps have been kindly placed at my disposal by the Swedish Geological Survey.

Finally, the darkest grey tone of the map indicates regions outside the alluvial valley, in the main the valley sides, where moraine and rock are exposed, or areas that lie quite outside the valley.

The preparation of the map and its accuracy

Before proceeding to the conclusions that may be drawn from the comparative map, we will first consider its preparation and accuracy.

The first stage in combining the different maps was to trace the Laga skifte map in its original scale, 1:4,000. Contours from earlier maps, and in some cases more recent maps as well, and finally from aerial photographs, were then inserted on this tracing. This was done with the help of optical projection (in a so-called Traut camera), which made it possible to correct for small differences in the scales of the maps due to uneven shrinkage, and for deviations from the normal scale of the aerial photographs (1:20,000). These drawings were then transferred to an enlargement of the original sheets (konceptblad) of the Generalstaben's topographic map to scale 1:20,000. Terraces, erosion marks, and hydrographic information were inserted in this composite map. The final copy was made to the same scale, after which it was reduced to scale 1:40,000 for reproduction. Examples of the Storskifte and the Laga skifte maps and of the aerial photographs are shown in fig. 35.

There are two requirements for accuracy in the present case: each of the surveys must give the position of the river shores exactly, and it must be possible to achieve a sure correlation of the maps from different years and the aerial photographs.

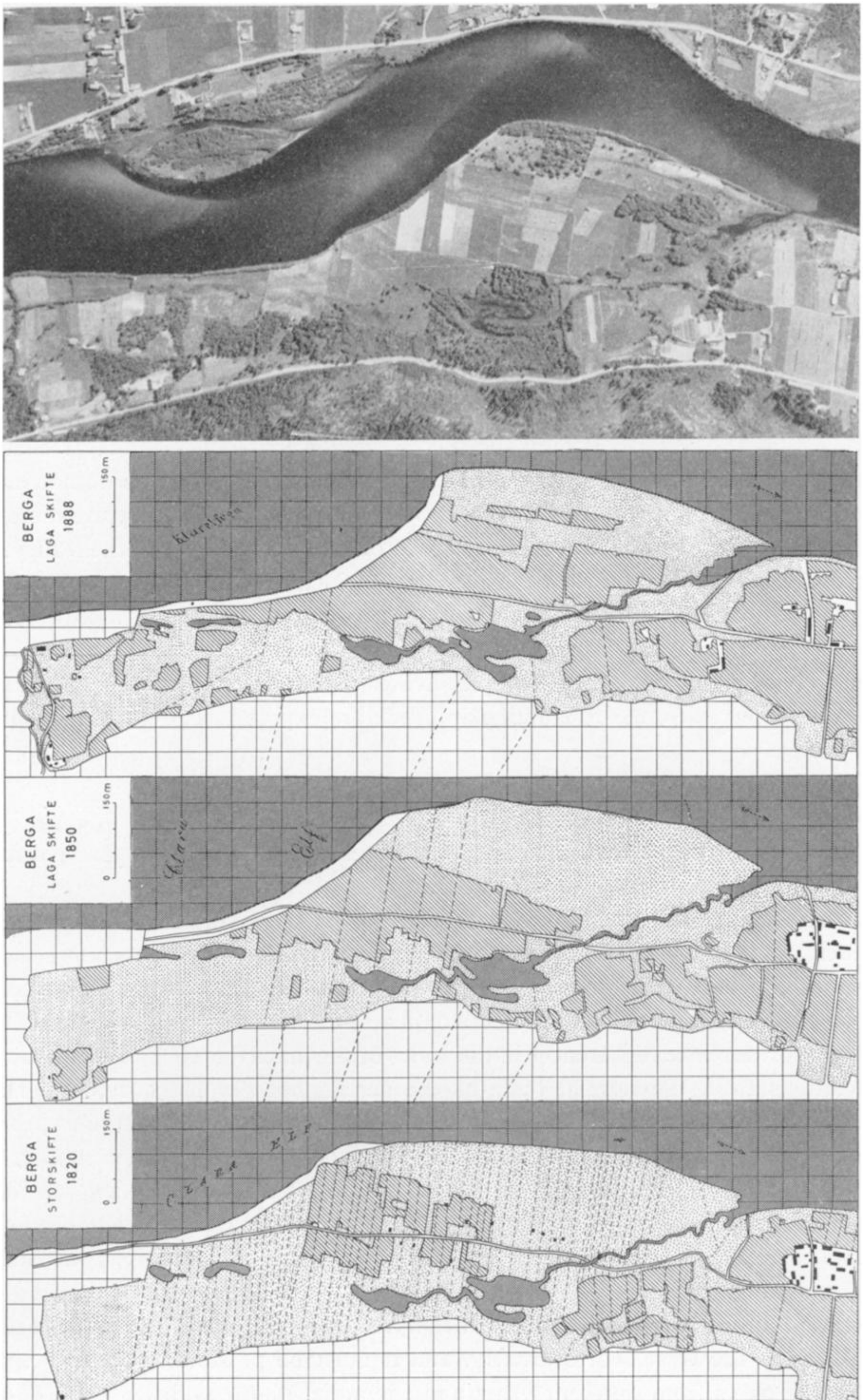

Fig. 35. The hamlet Berga in Ekshärad parish according to the Storskifte and Laga skifte maps and an aerial photograph. The square net is drawn to facilitate comparisons. Diagonally shaded areas: arable land. Dotted areas: pasture. White areas without squares: building sites and common land (älvbrott). The changes in the shore lines are distinctly discernible. Note the sand banks in the river channel.

It is obvious that the care taken in surveying different localities varied from case to case. It has already been mentioned that the maps from the seventeenth century and the early eighteenth were somewhat cursory, and therefore of variable quality. There are also quite large differences between the maps from the Storskifte and the Laga skifte. In places the maps are a reflection of the individual surveyor's competence and precision. However, the value of the particular piece of land was of greater importance than the personal factor in the surveying. A valuable piece of arable land was surveyed and divided with the greatest possible exactness, whereas less useful meadow land was treated with less care. Of great importance for the accuracy of the surveys is the fact that each headland in general was the site of a small village, which used the whole area of the headland as fields and meadows.

The peasants naturally set the highest value on building sites and arable land. Buildings and arable land were everywhere located on the higher northern parts of the headlands, which were more often spared from the spring floods, while the southern parts of the headland were pasture. In many cases the arable and the land immediately surrounding the farm buildings extended to the upstream side, where the shore was subject to erosion.

Thus the most valuable land was often near the eroded shore, where, though less liable to flooding, it was instead threatened by erosion of the river bank. In earlier times the people lived under a continual threat of crop-failure and famine, and they made use of every available patch of tillable soil to make their livelihood more secure: "... there are few farmsteads near the river with any plot of ground uncultivated, for those who live near the river have put every patch of ground to use, as there is little enough land between the hills and the river" ... (A. Mörner, Governor of the province, in his description of Värmland 1762).

It is evident that the river's inroads were a source of anxiety, and that attempts were made to hinder erosion by means of embankments and revetments. It was customary to mark off a strip of land bordering the erosion scarp to be used as common land (cf. fig. 35). For instance, in the protocol from the reallotment of land at Sälje in the parish of Ekshärad in 1763 we read: "Fifty ells of land belonging to the farm Nygård were set aside for the advance of the river, ten ells of the large field north of the farm, and on the southern side of the same field 50 ells; and it was agreed that this river strip (älfbrått) should be common land for the whole village, to be tilled by the tenants strip by strip."

In consequence of the value of the land near the erosion scarps and the risk of inroads from the river these parts of the headlands were some of the most accurately surveyed.

The downstream sides of headlands, on the other hand, were not considered so valuable. Each year the river rose high over the ground there, and when it withdrew left behind a great deal of sand and silt. The soil was water-logged, and useless except as rough pasture. "The pasture in the river valley is for the most part poor and scanty. It consists of only the lowest and swampiest parts along the shores of the river, which on account of the river's flooding cannot be used as arable ... For, when the river flowing there each spring, and sometimes even summer and autumn, floods these pasturelands, it flows so impetuously at high flood that it carries along and leaves behind on the pasture a great amount of sand, in some places half an ell, in some places a quarter, and in other places somewhat less" (MÖRNER 1762). In a few places, however, the land was regarded as of sufficient value to warrant a special division of the new land (e.g. Södra Loffstrand 1879).

Another factor may conceivably have affected the mapping of the flat low-lying land, that is, the river stage at the time of the surveying. Even apart from the extremes of high and low water it is not unusual for the water-level to rise or fall one or two metres in quite a short time in the meander zone of Klarälven. This does not matter as regards the shore line of an erosion scarp, but on the slight slopes of the low-lying land it means a displacement of the shore line by some tens of metres, sometimes even more.

However, a close study of a large number of maps reveals that the incidental stage of the river does not appear to have affected the maps. The limit of bush or tree vegetation seems to have been taken as the shore line, i.e. approximately the limit of the useful pastureland. This limit is in fact quite well-defined, and must be considered the proper shore line, regardless of the fact that it is occasionally flooded at high water and that extensive sand banks are visible on its riverward side at low water.

The correlation of the maps from the different periods and the aerial photographs usually presents no difficulty. As most of the land was under cultivation already at the end of the eighteenth century or beginning of the nineteenth, and as farming in these districts has maintained a conservative character up to the present time, the landscape has often been preserved even in detail. Village roads, boundaries between fields and between properties, in some cases housing sites and houses, and of course brooks, pools, and other topographical features, remain often almost unaltered from one map to a later one.

Examination of the shore line at places where it must have remained unaltered since the beginning of the nineteenth century provides an estimate of the possible error due to inaccuracy in the maps and in the procedure used to combine them. It may be briefly stated that the possible error appears to be of the order of 10 m, being rather less on the erosion scarps and rather more on the downstream side of the headlands. In the map as reproduced this signifies an error of 0.25 mm, and as finally drawn 0.5 mm. It is obvious that the error exceptionally may be greater in places at some distance from the central village or on land regarded as entirely without value.

Some results from the comparative map

It is immediately apparent from the map that the pattern of erosion on the upstream side of each headland and deposition on the downstream side is very pronounced, as expected. This is known to be a characteristic of many meandering rivers. Erosion damage seems to be on the whole greatest, at least as regards area, in the southern part of the meander zone. But there are notable local exceptions. It is therefore interesting to carry out a quantitative analysis of the map information in order to discover whether there have been any appreciable changes in the rate of erosion during the whole period, and whether such changes as may have occurred took place simultaneously over the entire meander zone.

This analysis has been carried out in the following manner. The entire meander course between Vingängsjön and Edebäck was divided up into 10 subsections, each 10 km long. The area eroded on each headland was measured on the original combined map of scale 1:4,000, for the two periods between successive maps. The area thus obtained was then divided by the number of years elapsing between the surveys. The result is the average annual erosion for each headland during the respective periods.

As already mentioned, the maps and the aerial photographs may be referred to three years: 1800, 1850, and 1950, and consequently the rates of erosion can be divided into the two periods 1800—1850 and 1850—1950. This division is not precise, of course: for instance, if a headland was surveyed in 1847 and heavy erosion took place in 1849, this erosion will be included in the later period although it actually occurred in the earlier. However, it is impossible to avoid such small displacements in time, and it is not likely that they have any appreciable effect on the result of the analysis.

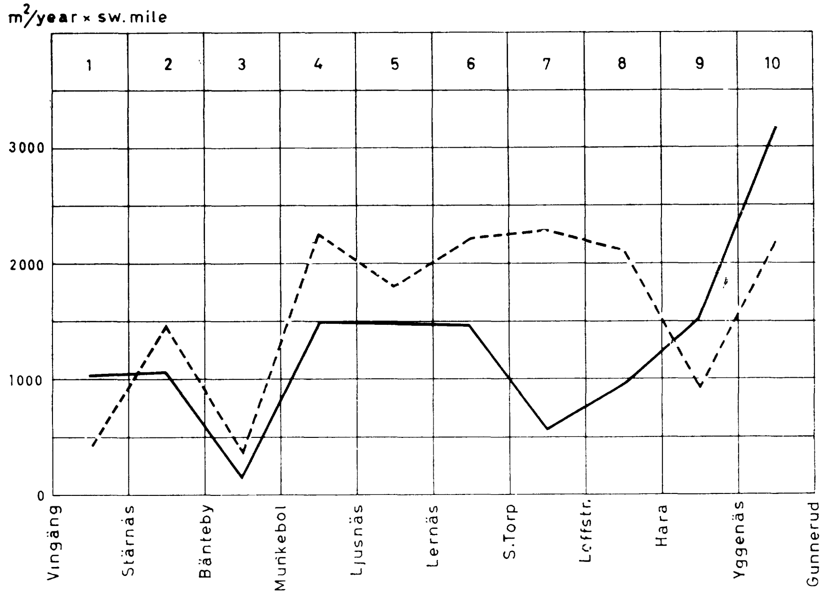

Fig. 36. The rate of erosion in different parts of the meander course. Dashed curve: average area eroded per year and per Swedish mile (= 10 km) during the period 1800—1850. Continuous curve: average area eroded per year and per 10 km during the period 1850—1950.

Fig. 36 shows the results in diagram form. The dashed curve denotes the average area eroded per year and per 10 km during the period 1800—1850, the continuous curve the corresponding area during the period 1850—1950.

For *the entire length* of the meander course the eroded area was 16,000 m² per year during the first period, and during the second 12,800 m² per year. This suggests a slight decrease in the erosional activity, but the difference is so small (20 %) that it cannot be considered significant. There are some interesting features in the shape of the curves, however.

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The general tendency of erosion damage to increase southward is clear from the diagram, being clearly marked for both periods. For the greater part of the meander zone the erosion damage is less during the second period. The exceptions are the most northerly subsection, and, even more so, the most southerly one.

It is probable that a special circumstance led to the accelerated erosion in the southern part of the course. Sometime between 1820 and 1840 the loop north of Sälje was cut off by the river.[1] The transport of sediment naturally increased considerably in the region downstream from the cut-off after this. The attack on the erosion scarps intensified, especially at Ginbergsängen.

This is in all probability one of the reasons that Ginbergsängen experienced by far the largest loss of land throughout the meander zone during the past hundred years. Altogether 160,000 m^2 of land were lost during the period. The rapid erosion was favoured by the circumstance that the erosion scarp is easily erodible, and unusually low, and that there was no protection whatever against erosion. The measurements of the position of the erosion scarp that have been regularly carried out during recent years show that the river continues to be as active as before. Since the land is low there, and even lower farther south, this headland will eventually be completely cut off, unless measures are taken to prevent it. If a cut-off occurs, it is obviously important to take the necessary steps in good time to meet the consequences in downstream regions.

Another noteworthy circumstance is the slightness of the erosion in the section between Bänteby and Munkebol. The reason is that the river there in many places runs over moraine and erosion-resistant glaciofluvial material, which probably originated to a large extent in the drainage system of the tributary river Femtån.[2] This has caused the river channel to become more or less fixed, thus disturbing the regular pattern of meanders. Minor revetments also occur there, and they have helped to stabilise the river channel.

Apart from the cut-off at Sälje there has only been one real cut-off during the whole period. In the meander loop between Mjönäs and Gravolsmon the river cut off the tip of the southern headland during the catastrophic flood of 1916, and a small island was formed. (See Pl. I.)

We will not undertake any detailed discussion of the causes for the great differences in the rate of erosion in the various bends. The process and the causal connections will be clearer after the section on "The flow conditions and their significance for the processes of erosion, transport, and sedimentation" (p. 281). Here we will only consider some general aspects and conclusions.

The variation in the erosion-resistance of the erosion scarps is one of the main causes of the differences. The most easily eroded material is loose alluvial sediment in the size

[1] According to the annual report of Kungliga Wermeländska Hushållningssällskapet for 1818, Olof Olsson from Stångerud, member of the Riksdag, applied for a grant from the Government for, among other things, the digging of a canal through a headland lying between the villages Nore and Sälje in order to diminish the effect of the river floods. According to notes in BÄCKVALL's Elfdalsarkiv the canal was also actually dug. For a long time it was known as "the canal". This operation probably hastened the cutting off of the meander loop.

[2] This was already pointed out by DE GEER (1911, pp. 22 and 55).

ranges sand and silt. Where coarser material is also present, glaciofluvial material or moraine, the resistance to erosion increases. Vegetation, especially bushes or small trees, also raises the resistance. Artificial protection against erosion is effective for a limited period. If the eroded slope is high, the process of erosion may be hindered by the large amount of material that slides down into the river as the river undermines the slope.

The other main cause of the local variations in the rate of erosion is differences in the manner in which the current impinges on the river bank, in the velocity of flow, and in the state of turbulence. The nearer the main current approaches the erosion scarp the greater the flow velocity becomes near the side of the channel, and consequently the greater the eroding power of the water. The general appearance of the river channel, the curvature of the meander bend, and the morphological details of the river bed are therefore important erosion factors, in so far as they determine the state of flow near the erosion scarp.

The varying water discharge from year to year, alterations in the shape of the river bed, and alterations in the structure of the erosion scarp are conditions associated with large variations in the yearly rate of erosion. Several years may pass without serious damage occurring on a meander bend, but then suddenly the river can carry off some metres of its bank during a single year or during a single flood period. Generally such activity is due to an especially heavy flood of water, but sometimes there is some other reason. A revetment may have been demolished, or the current may have been diverted in a disastrous way.

An instance of the present erosive activity

An instance of the initiation of an intensive period of erosion is provided by developments on Götnäs in Ekshärad parish. In 1860 the shore line was relatively even from the upper corner of the headland to its tip (see pl. I). Subsequently the river has cut into the upper part but remained fairly stationary in the middle part. The result may be seen in the aerial photograph from 1950, where a deep incision may be seen in the north of the headland. The reason that erosion was slowed down in the middle part of the erosion scarp is the presence there of a projecting mound of erosion-resistant glaciofluvial material with boulders up to 60—100 cm. This esker has evidently been sufficient to halt the lateral erosion on this part of the river bank.

However, at the beginning of the 1950's the river began to overcome the obstacle. The present position of the glaciofluvial material is shown in the sketch map on the upper left of fig. 37. The stones and boulders appear above the water only at low water, and are completely covered at higher levels of the river. The photograph on the left in fig. 38 was taken towards the erosion scarp at low water, and the projecting stones are visible on the right.

At high water the esker now gives rise to heightened turbulence; strong eddies are formed, and part of the flow is deflected in against the erosion scarp. Thus, instead of being an obstacle to erosion as earlier, the esker has now begun to assist the eroding forces. And erosion of the adjoining bank has in fact proceeded quite rapidly during the past few years. Three recent measurements of the position of the erosion scarp are given in fig. 37. The bank, which is unprotected, has receded 5—10 m along a front of about 75 m.

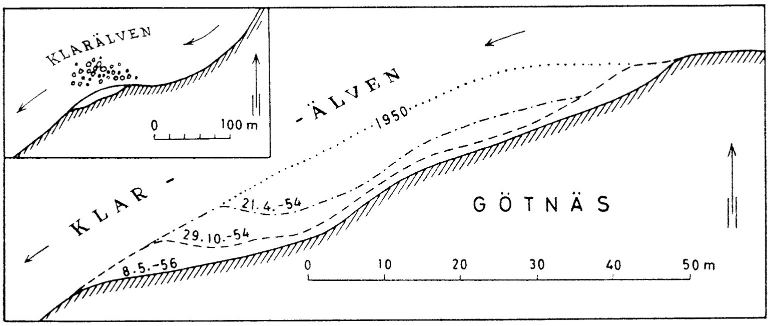

Fig. 37. Changes in the shore line along the erosion scarp at Götnäs in Ekshärad parish according to recent measurements. The position of the temporary obstacle of glaciofluvial material is marked by small rings in the sketch on the upper left.

Since the risk of a breach in the river bank was considered to be serious if erosion continued at Götnäs, a revetment has been built along the inner part of the cavity upstream from the esker. The photograph on the bottom right in fig. 38 shows this revetment under erection. Further reinforcement of the bank will probably be necessary as soon as a particularly heavy spring flood occurs.

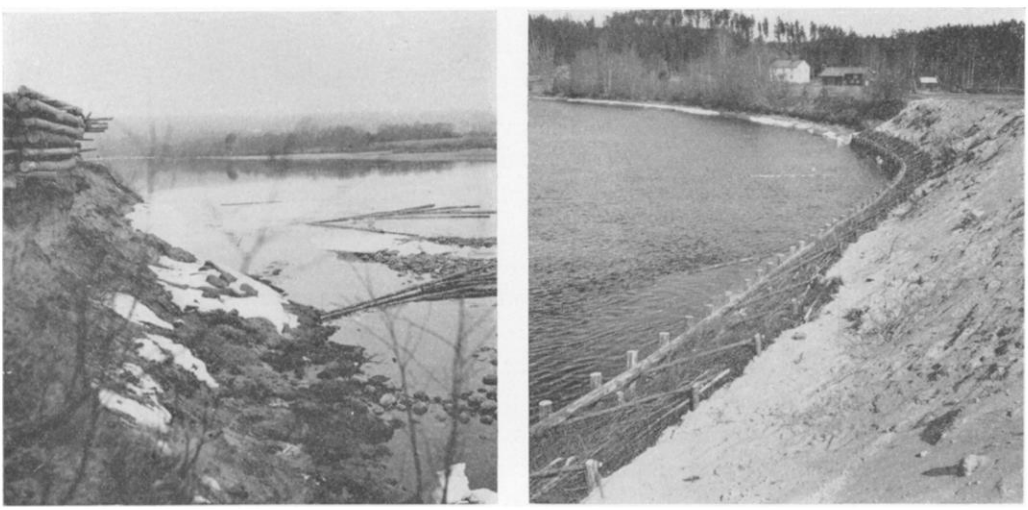

Fig. 38. Left: part of the erosion scarp at Götnäs with the coarse glaciofluvial material projecting above the water level to the right. Right: revetment under erection at Götnäs (April 1954).

Recapitulation

To summarise the results of the above investigation, it may be stated that the rate of erosion in the meander zone of the upper Klarälven manifests large variations, both in time and with respect to the various meander bends. The examination of old records and study of comparative map material both show that erosion has caused appreciable damage as far back as the records go, i.e. at least 400 years. *In spite of the many accurate maps from the nineteenth century it has not been possible to trace any definite trend in the rate of erosion, though there is perhaps a slight hint of a decrease in erosion damage. The average loss of land since the beginning of the nineteenth century has been about 13,000 m^2 per year. The historical development gives no confirmation for the view that there is reason to expect a considerable change in the progress of erosion in the future, unless there should occur some marked change in the external conditions.*

A study of the old maps from around 1650 does not reveal any drastic changes in the period from 1650 to 1800. *All abandoned meander loops except those at Sälje and Mjönäs* can be regarded as older than 300 years.

THE MORPHOLOGY OF THE VALLEY FLOOR AND OF THE RIVER BED

The morphology of the meander headlands

General

The term *valley floor* is here used to denote the part of the valley which is lower than the terraces of the valley sides. Thus defined, the valley floor more or less coincides with the lobes of land that jut out alternately from the western and eastern sides of the valley. A description of the morphology of the valley bottom is therefore a description of the morphology of these meander headlands.

There are about seventy clearly defined headlands between Vingängsjön and Edebäck. Their characteristic morphological feature is that they are entirely made up of point bars, which usually run parallel with the downstream side of the headland. Since the height of the point bars gradually increases with distance from the shore line, the headland resembles a slightly tilted, triangular, corrugated surface, with the ridges more or less abruptly truncated at the relatively high erosion scarp on the upstream side. On many headlands there are formations, e.g. ox-bow lakes, which disturb the regular morphological pattern.

The Stärnäs-Lillängen headland

Pl. II is a large-scale map of the Stärnäs-Lillängen headland in the parish of Dalby (cf. Pl. I)[1]. The map has been drawn in the crayon manner, and in addition has height contours at 0.5 m intervals. The map really encloses three headlands, which have been united by the cutting-off of a meander lobe. The abandoned loop is by no means filled with sediment, although the cut-off must have occurred at least 300 years ago.

[1] The survey was made and the map was drawn by Å. FALK as examination task for the degree of filosofie magister, at the suggestion of the author. In a coming number of Geographica (Papers from the Geographical Institute, Upsala University) FALK will give a detailed description of the map and the headland, together with an account of the historical development of the headland.

The morphology of the region is typical of the whole meander course, and therefore a short description of the map is not out of place (in the main FALK's manuscript is followed).

The northern part of the headland is a regular series of point bars with the crests of the ridges up to 3 m above the troughs. The ridges are asymmetric in their transverse section: the steeper side (25—30° slope) faces southward, and the other (5—10° slope) northward. The point bars flatten out towards the valley side, but there are some irregular sharp-topped ridges near the valley side and parallel to it. In general there is no direct connection between these ridges and the regular point bars. The terrain between the irregular ridges and the point bars is low-lying and marshy, and there is a belt of small elongated pools, the *lagoon zone* as DE GEER has called it. The general slope of the land on the headland is c. 1: 100, in a direction down the valley and slightly in towards the side of the valley. This is typical of the morphology of the meander headlands.

While the northern part of the headland is rather near the western side of the valley, the middle part is in the centre of the valley. The morphology of this part is therefore rather different. The point bars are smooth arcs convex down the valley, unlike those of the northern part, which are convex up the valley. Also, the transverse section through the ridges is more symmetric, though the steeper side is still the southern one. Some of the ridges can be traced right across the headland, but many of them disappear before they reach the eastern side. The average distance between the ridges is here c. 17 m, as compared with 27 m in the northern part of the headland. There is a vestige of the lagoon zone here as well, but the zone proper was probably eradicated in connection with the cutting-off. The general slope is slight (c. 1 : 300).

The southern part is the remnant of a very narrow headland, which prior to the merging of the three parts was reduced by erosion on the upstream side without compensating deposition on the downstream side. This part consequently lacks the regular point-bar formations of the other parts. However, there are some distinct ridges, and the lagoon zone is relatively well defined.

The whole headland is flooded at extreme high water. The level of the flood water during the catastrophic flood of 1916 attained about 151 m above sea level, i.e. 6.5 m above the level of the shore line of the map, and about 7.5 m above the normal low-water level (cf. FALK 1953). The large pool in the middle of the southern part of the headland was excavated by the river in 1916.

The height of the headlands

A comparison of headlands in different parts of the meander zone reveals that the height of the highest point of each headland with respect to the nearby water level decreases on an average from north to south. However, the decrease is not quite regular. For instance, the Stärnäs-Lillängen headland just described is rather low. The erosion scarps are in general about 8—10 m high in Dalby and Norra Ny, and in some places up to 12 m. Farther south in Ekshärad the height is in general 3—8 m, sometimes less (cf. DE GEER, p. 29).

Since the point bars developed while the river was undergoing a gradual lateral displacement the general slope of the headlands must be mainly due to a progressive vertical erosion, as DE GEER has pointed out. The level of the river at Edsforsen immediately downstream from Edebäck must have been the same for several thousand years. Hence the downcutting must have been most marked in the northern part of the meander zone, a conclusion which accords with the greater height of the erosion banks in the north.

DE GEER's opinion (pp. 164—5), though not fully clear, seems to be that the vertical erosion proceeded by the breaching of local base-levels of moraine or glaciofluvial material and by a continual approach of the river's gradient to an equilibrium value.

But it is uncertain whether there has in fact existed any local base-levels above Edebäck during the postglacial period. Moreover, in seeking to relate the height of the erosion scarps along the river to the vertical erosion, other factors must be taken into consideration. The fact that the height of the headland increases towards the north is due to a combination of circumstances.

1. *The unevenness of the land uplift* in Scandinavia is raising the northern part of the meander course faster than the southern part. The difference of land uplift between Vingängsjön and Edebäck at the present time appears to be about 10 cm per century, or 1 m per thousand years (interpolation of the values given by BERGSTEN 1954, p. 106). Though rather uncertain as to its numerical value, this difference is so large that it must be an important contributory cause of the vertical erosion in the upper parts of the zone.

2. *Changes in the run-off conditions* may have led to more active vertical erosion. An increase in the volume of flow has two effects: the flow velocity increases, and the depth of water increases. These factors lead to greater vertical erosion, and tend to decrease the curvature of the meander loops, so that they straighten out and shorten the course of the river.[1] A shortening of the river's course in its turn leads to downcutting in the upper part of the meandering course. There are morphological indications of such a change in the water discharge during relatively recent postglacial time. Most of the oldest discernible cut-off meander loops have a curvature and a transverse section that would not accommodate the present volume of flow at high water (see Pl. I). For instance, the small extinct loop on Uggenäs in the parish of Dalby is considerably smaller than any meander loop of the present river.

For comparison it may be mentioned that SANDEGREN (1939, pp. 22—23) has been able to trace a straightening of Klarälven's course in the deposits of the former delta at Grava just north of Karlstad, and he attributes it to an increase in the water discharge due to climatic change. Similar observations have been made in the meander zone of Österdalälven north-west of Mora by WENNER and LANNERBRO (1952, p. 89), though they ascribe the increase in the peak flows mainly to river captures and changes in the channel storage.

This interesting problem has not been studied in detail, so it is not possible to put forward more than a surmise that the volume of flow in the meandering Klarälven was

[1] The shortening of the river's course with the decrease of the curvature of the meander loops is mainly due to the prevention of the free development of the meander loops by the valley sides in this particular case.

at some time, probably prior to the deterioration of climate about 500 B.C., less than at present, and the course more winding, less restrained by the valley sides. In consequence of the more winding course the total distance travelled by the water was then larger, and as the gradient must have been just as great or greater, the difference between the levels at Vingängsjön and Edebäck must have been larger.

A yet more drastic straightening of a meandering course appears to have taken place in the case of the river Glomma in Solör in Norway, according to investigations by FALCK-MUUS (1953, p. 18). It may be possible that the change there also has been an effect of the deterioration of climate.

3. *The more rapid lateral erosion in the southern part of the meander zone* implies that the average age of the headlands there is lower. For, the more rapidly erosion proceeds, the younger the deposits must be which are cut away from the erosion scarps, and the shorter the time during which the river has cut down since the deposits were laid down. Younger headlands are thus lower than older headlands.

4. *The large variations in the water stage in the northern part of the meander zone* enables the river to build up higher point bars there than in the more southerly part of its course. However, this effect is perhaps counteracted to some extent by the lower transportation of sediment in the north.

Factors 1 and 2 both lead to increased downcutting, 3 allows vertical erosion to continue for a longer time, and 4 means that the point bars can be built up higher in the upper part of the meander zone. All four factors have no doubt contributed to the greater height differences in the northerly headlands.

The "näsavsats"

A morphological phenomenon discussed by DE GEER (see, for instance, p. 162) is intimately connected with vertical erosion. It is the step which sometimes occurs in the topography of a meander headland, with higher point bar ridges on the northern side and lower ridges on the southern side (DE GEER's term is *näsavsats*). The height of the step is seldom more than 4 m. DE GEER supposes (p. 163) that the step is due to a catastrophic downcutting activity that developed when the level of the river suddenly fell some metres in consequence of the breaching of local base-levels.

According to DE GEER's hypothesis these steps should have formed at the same time along a fairly long stretch of the river, since pronounced vertical erosion cannot occur locally. A fact which does not support DE GEER's explanation is that steps of varying height are encountered at quite different levels and at all conceivable distances from the present sites of sedimentation. For instance, the declivity on the headland Uggenäs north of Stärnäs on FALK's map (see Pl. II) may be regarded as one of these steps; likewise the higher step on the southern part of Stärnäs-Lillängen. In the former case the step must have been formed when the cutting-off led to a phase of rapid lateral erosion and curtailed deposition above the cut-off following on a preceding phase of relatively stable shore lines and continuous, slow downcutting.

The above explanation of the connection between vertical erosion and the height of the meander headland means that the formation of a step is also associated with the relative intensities of vertical and lateral erosion. When the river channel is stationary, the river tends to entrench itself as downcutting proceeds. There is also time for the point bars to rise to a level near the highest water stage, or at least above the normal high-water stage. On the other hand, when lateral erosion proceeds at an appreciable rate there is no time for vertical erosion to have any definite effect during the time that elapses between the development of two point bars. The more rapid the lateral erosion the lower the point bars. A typical feature of meander lobes where the process of erosion has proceeded rapidly is that there are as a rule large lagoon belts in the unfilled parts near the side of the valley, as an examination of Pl. I and the aerial photographs will show.

The explanation of the step formations advanced above implies that these formations are of little use for a reconstruction of the river's course at some previous time, unless it can be shown that steps on neighbouring meander headlands developed during a phase of renewed lateral erosion along a considerable stretch of the river.

The important question of how the point bars have been formed will be taken up in a subsequent section (p. 288).

The morphology of the erosion scarps

General remarks

Since the development of a meandering river, with its more or less continual lateral erosion and lateral displacement of the channel, is accompanied by a gradual retreat of the erosion scarps, an investigation of these scarps is an important part of the study of fluvial formations and processes.

As previously mentioned, the height of the erosion scarps along the meanders of Klarälven varies from 3—5 m to 10—12 m. This disregards the few places at which the river runs in against an old terrace, where the height of the slope may be considerably greater. An example of such a place is the northern corner of Ändenäs in Norra Ny parish (fig. 40), where a remnant of a terrace is 28 m above the average water level of the river, and where active erosion is now going on.

The morphology of an erosion scarp is of course determined by the stratification and grain-size distribution of the bank material, by vegetation, by any structures that may have been built to protect the bank, and by the nature of the destructive processes.

The stratigraphy and composition of the soil will be described in the next section. Here it suffices to point out that the sediment in an erosion scarp is normally sand and silt, and thus very easily eroded. Only in exceptional cases does coarser erosion-resistant material such as coarse gravel or boulders occur. But where the river washes against the side of the valley the shore is generally composed of coarse resistant material or even rock.

Where there is no coarse material in the erosion scarp it is mainly the stratification and component grain sizes that determine the slope. In fine-grained material of sizes less than about 0.6 mm the scarp is often steeper than the normal angle of repose, especially when the moisture content is high, since the cohesive and adhesive forces outweigh the gravitational.

The occurrence of fine-grained, cohesive material also affects the manner in which soil moves down into the river, and thus indirectly influences the shape of the scarp. Typical frictional material rolls or slips down the slope, and the individual particles move at least to some extent individually. The slope of the bank is then maintained relatively unchanged, and is determined by the angle of repose for the material in question. In cohesive material, on the other hand, it is usually larger aggregations of particles that move: a *slide* or *slope failure* occurs, leaving typical slide scars. There are naturally forms intermediate between these two.

Vegetation usually helps to hold loose soil together and retard the destructive process. Where erosion is active the top of the scarp is usually overhanging, held together by vegetation. Small trees and bushes with well-developed roots often provide some protection against erosion for a limited period.

The intensity of the local erosion is reflected in the morphology of the erosion scarp. Where the bank is particularly exposed to the action of the river vegetation cannot take root. The erosion scarp is then bare, and often steeper than the angle of repose for the material there. But when the erosion is slower the slow attritional processes have time to smooth out the slope, and it is possible for vegetation to obtain a foothold and convert the slope to a stable, consolidated bank without a trace of active erosion.

Owing to the variation of the intensity of erosion during the year the appearance of exposed erosion scarps changes with the time of year. During a period of marked river activity or immediately afterwards the slope often becomes steeper, whereas after a period of lesser activity it is smoothed out.

Description of some localities

After these general remarks, based on studies of all the erosion scarps in the area, we may pass on to particular instances illustrated by photographs from some representative localities.

The photograph on the upper left in fig. 39 was taken in the upstream direction at Krusmon in Norra Ny parish. It illustrates a regular erosion scarp where erosion is active. The scarp is about 12 m high and practically free from vegetation along its entire length, which stretches some 700 m. There are only occasional small bushes and tufts of grass, of which the majority have slid down from the crest of the slope and are slowly progressing towards the water. The top of the slope is overhanging, the soil there being bound together by vegetation. Below the overhang the slope is about 30—40°, but decreases slightly towards the surface of the river. The slope is thus close to the angle of repose. The material is almost entirely sand.

The photograph on the upper right is of the erosion scarp at Ljusnästorp, also in Norra Ny parish. There is an overhanging edge held together by vegetation, but also in the strata immediately underneath the slope is almost vertical, on account of the water-retaining properties of the sediment (median grain size c. 0.1—0.2 mm).

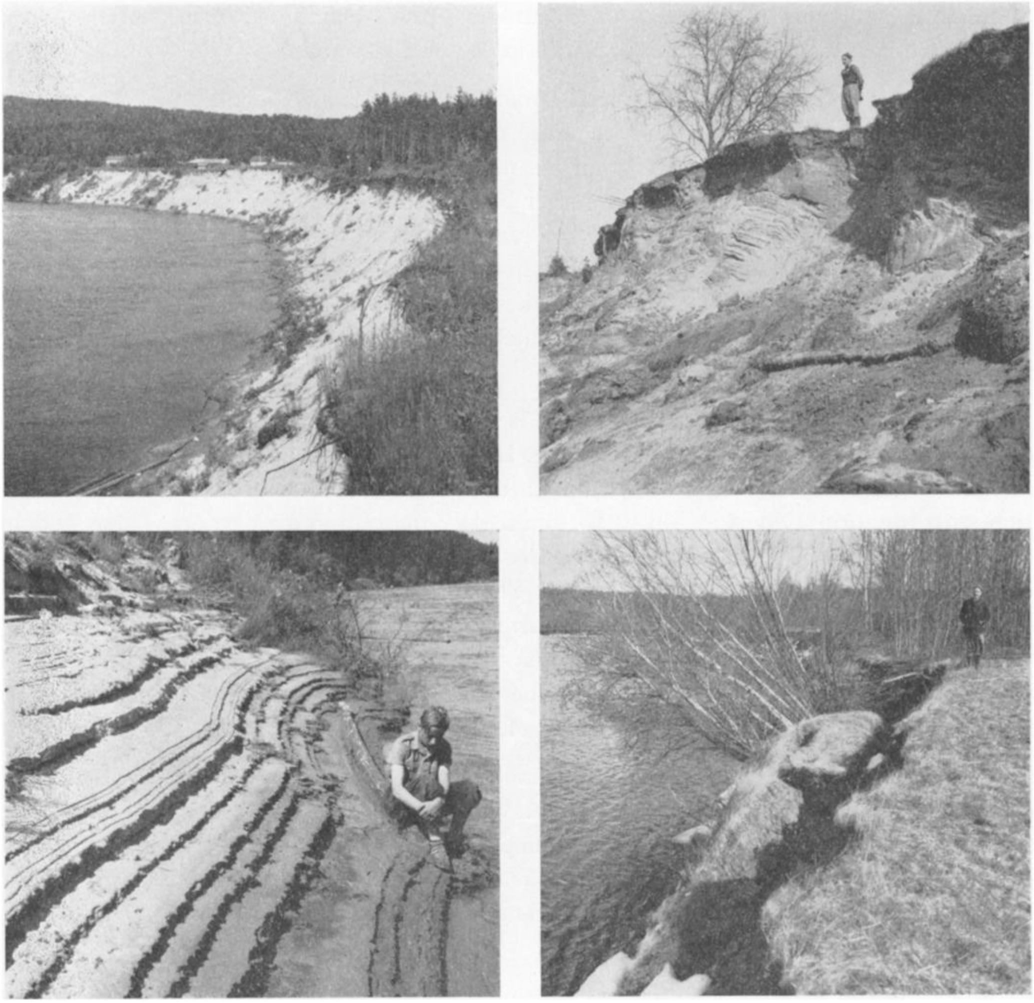

Fig. 39. Upper left: the erosion scarp at Krusmon looking upstream (July 1952). Upper right: part of the erosion scarp at Ljusnästorp (July 1953). Lower left: miniature terraces in the erosion scarp at Stärnäs (June 1952). Lower right: block of soil with bushes and small trees sliding down into the water on the erosion scarp at Ginbergsängen (April 1954).

The strata in an erosion scarp are sometimes etched out by the wind, which removes material from the unprotected parts of the slope, carrying the finest fractions in over the adjoining land. The vegetation and ground near an erosion scarp is in consequence often covered with a very thin layer of eolian sediment, but this layer does not extend far inland, and is of no significance as regards the soil structure.

The photograph on the lower left shows miniature terraces in the erosion scarp at Stärnäs in Dalby parish. The heavy erosion during the spring flood often creates an indentation at the highest water level. Subsequently, as the water gradually falls, series of miniature terraces are formed at favourable places, mainly by wave action. Wave erosion is otherwise of no great importance in the meander zone.

The photograph on the lower right is from the low meander headland Ginbergsängen in Ekshärad parish. At high water there the river reaches the top of the scarp, and even higher. The water thus acts directly on the material comprising the bank, which is fine-grained, in part cohesive, and held together by vegetation. The result is that large lumps of the bank break away. These slide gradually down into the water, where they are eventually broken down and removed. Groups of bushes and small trees are often carried along with the blocks of soil. It usually takes several years for such a block of material to be washed away by the river, but in some places the process is more rapid, especially when there are heavy floods.

Erosion along the meanders of Klarälven is little affected by ice, which elsewhere in this climatic region is such an important factor. In many of the rivers of Norrland, and even in some other parts of Klarälven, damming and erosion damage are common in connection with the formation of ice-slush, bottom ice, ice dams, and ice barriers. These seldom occur in the meander zone. However, the freezing of the river and the break-up of the ice do affect the river banks to some extent. The photograph on the upper left in fig. 40 shows an ice-floe still attached to the bank after the break-up of the ice. But it is beginning to break away, and in so doing it will loosen a good deal of soil from the bank. The load of sediment increases at such times, and the occurrence of the ice break-up can often be traced in the curves for the concentration of suspended material (cf. p. 300).

When the snow melts and the ground thaws out there is often strong activity in fine-grained soils that are liable to solifluction. Water-saturated soil often flows over snow fields lower down, forming tongues resembling miniature streams of lava. The photograph on the upper right is from a ravine in a terrace near Baskenäs in Norra Ny parish, but similar formations often occur on the erosion scarps along the meandering river during the spring.

Ravines are in process of formation at some places in the valley of Klarälven. These places are generally located on the higher terraces of the valley sides, where the material is more fine-grained. The terrace slopes are also often furrowed by old ravines where there is now further activity. A fine example of a recent ravine is to be found in the northernmost part of Ändenäs in Norra Ny parish. A small stream, which is sometimes dry, crosses a terrace about 28 m high just before its oblique confluence with Klarälven. The terrace is thus subject to erosion from two directions, and the bare edges of the terrace meet in an irregular sharp edge (photograph on the lower left).

There are no real landslides in the meander zone, except along the terraces of the valley sides. On rare occasions the fine material there may slip in a slide of considerable dimensions when the stability of the slope is disturbed. The steep sides of the terrace at Ändenäs have arisen in this way. But soil movement during the spring and during persistent rain are gradually smoothing out the steep slope (photograph on the lower right).

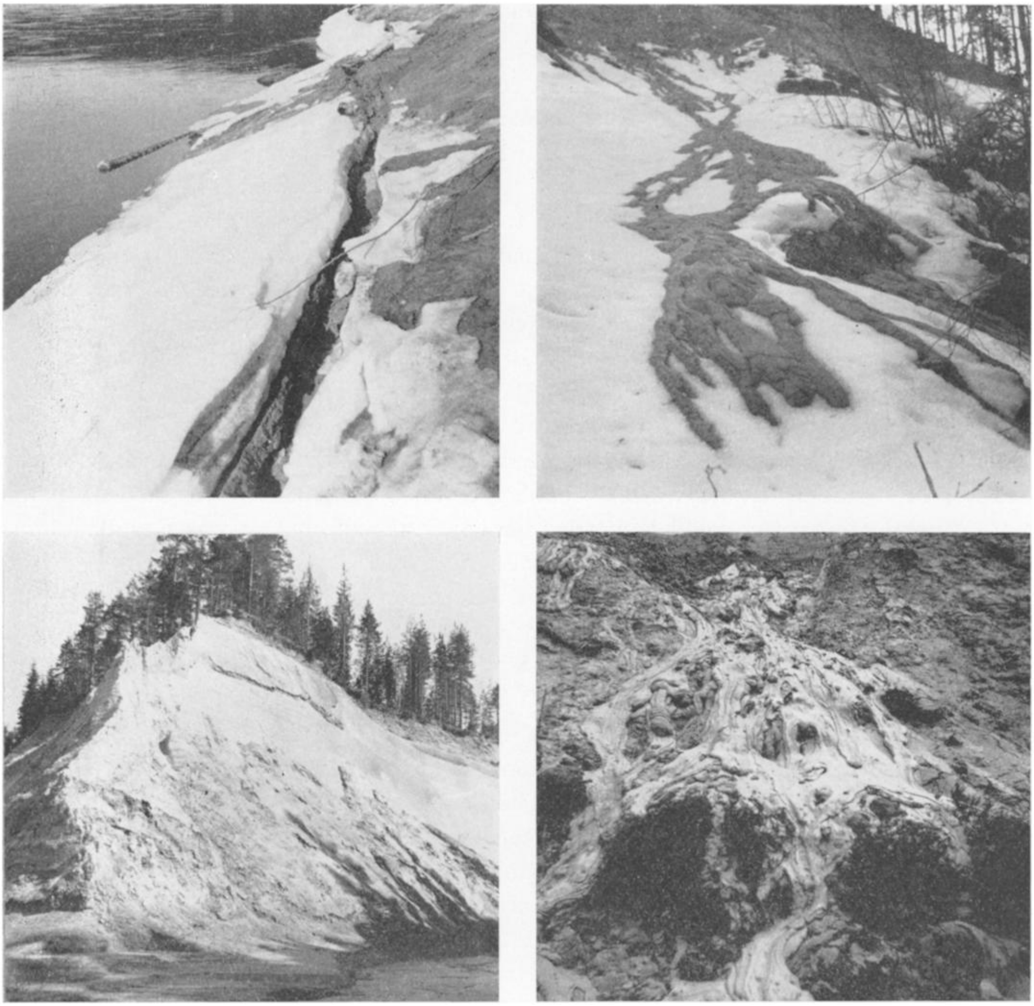

Fig. 40. Upper left: ice-floe breaking away from the bank (Götnäs April 1954). Upper right: tongues of water-saturated, fine-grained soil flowing down the sides of a ravine near Baskenäs during the thaw period (April 1954). Lower left: the terrace edge at Ändenäs. Note the coarser river sediment overlying the fine-grained fjord sediment (July 1952). Lower right: fine-grained soil flowing down the slope after heavy rain on the Ändenäs terrace (July 1953).

The morphology of the river bed

The meander course from Munkebol to Värnäs

As may be seen from Pl. I, the course of the river is quite regular within certain parts of the meander zone. The dominating pattern is a regular oscillation from one side of the valley to the other, with relatively slight curvature on the stretches between the valley sides, but a sharp, almost right-angle bend where the river impinges against the sides of the valley. However, some stretches are rather irregular, and in the south the river is fairly straight from Rudsängen to Edebäck, with only a slight tendency to meandering.

The most regular part of the meandering course is from Munkebol to Värnäs in Norra Ny parish. This was the stretch of river chosen by De Geer for detailed study. One result is a detailed map (De Geer 1911, Tafla I), with depth contours at 1 m intervals, for the stretch from Björkenäs to Värnäs.

As part of the author's investigations in the valley of Klarälven, the same stretch of river, a little over 7 km long, was again sounded on sections 100 m apart, and with a distance of 5 m between adjacent soundings. The aim of the measurements was to find whether there had been any marked changes during the past 45 years. The local depth of a river is subject to continual change as the water discharge rises and falls, and as sand banks slowly move downstream, but such short-term changes in the morphology of the river bed were not considered in the present comparison.

Unfortunately, it was found that the geodetic measurements of the previous survey had not been sufficiently accurate to enable a detailed comparison to be made. But the more general comparison could be carried out, and, as expected, it was found that *the main features of the bed had remained practically unaltered.*

Part of the river stretch where soundings were made is shown on the left in fig. 41. The depth contours are at intervals of 1 m, and different depth intervals are indicated by different shades. The reference level for depths was taken as a water level of 142.00 m above sea level at the river gage at Stöllet. The contours in the areas with the darkest shading have unfortunately been obscured during reproduction.

The varying width of the transverse sections is immediately apparent. Where the river impinges against the erosion-resistant material of the valley side the river is narrow, and the bed is deeply excavated. Off the tip of Baskenäs at the western edge of the valley the river is 70 m wide and a little more than 10 m deep; at the tip of Ändenäs the corresponding figures are 66 m and 11 m, and at Ljusnästorp 81 m and 10 m.

On the slightly curved stretches through the easily eroded sediment in the middle of the valley the river is much wider—between Baskenäs and Ändenäs reaching a maximum of 148 m, and between Ändenäs and Ljusnästorp 194 m. The transverse section is there markedly asymmetric, with the greatest depth close to the erosion scarp on the southern side of the channel (cf. also fig. 55). The maximum depth at the prevailing water stage was not more than c. 4 m.

On the downstream side of each headland there is an extensive shallow region, which rises above the surface at low-water, revealing a large sand bank covered with transverse bars and ripples (cf. fig. 45). Between this sand bank and the headland there is sometimes a narrow channel, of a depth always much smaller than that of the main channel. This subsidiary channel is especially prominent when the river is very wide, and when the current is deflected from the valley side by some obstacle there. An instance is the eastern river bend in the figure.

The large variations in the width and depth of the river are obviously due to the disturbances of the flow caused by the sudden bends at the sides of the valley. Turbulence

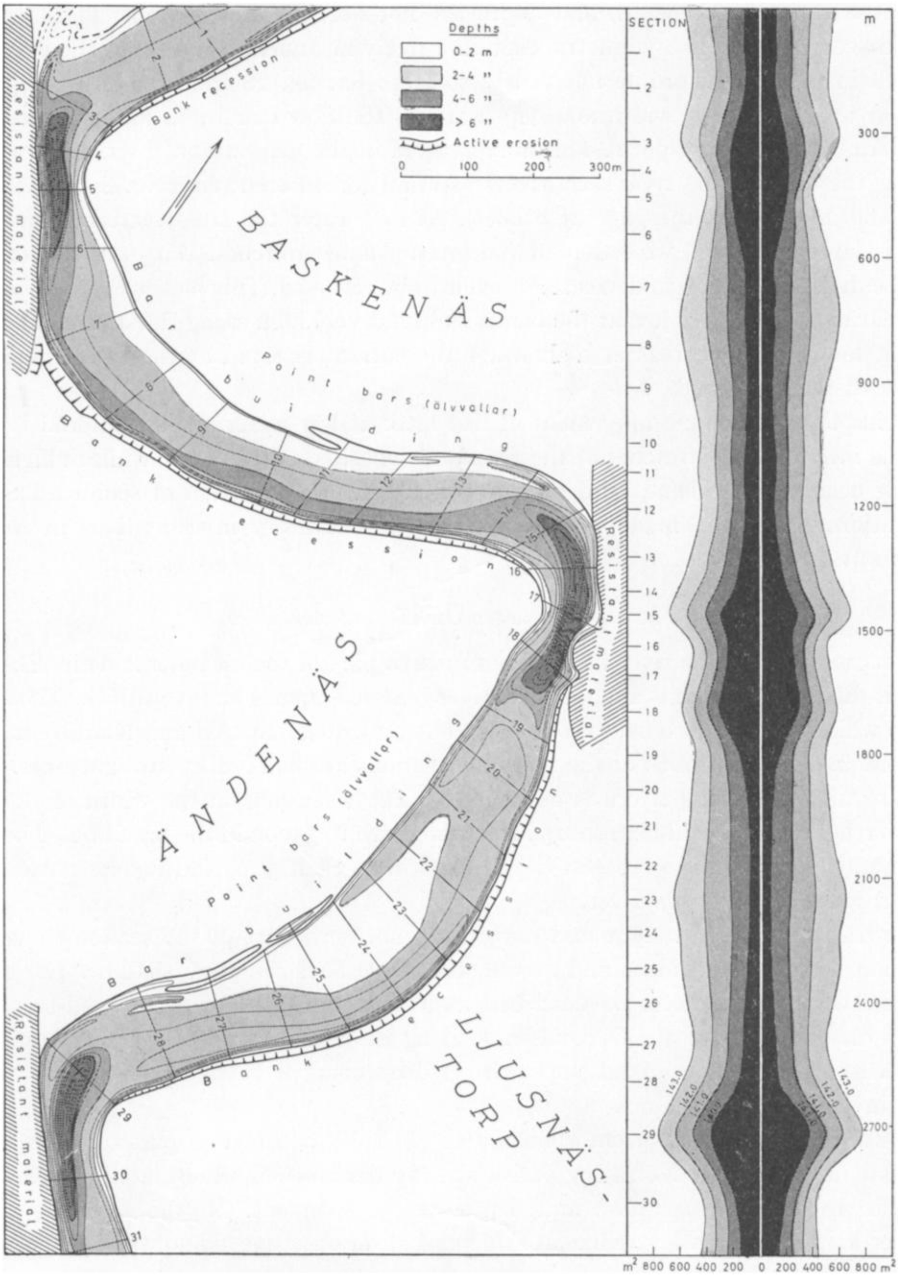

Fig. 41. Map of the river between Björkenäs and Ljusnästorp in Norra Ny parish. Depth contours at intervals of 1 m. The diagram on the right shows the cross-section areas for different river stages. The section numbers are marked on the left in the diagram. The position of the sections is shown on the map.

and eddies are strong there, especially at high water, and a deep narrow hole is excavated. Such large variations in width and depth are not normal for freely meandering rivers. The transverse section is asymmetric even in a freely meandering river, but the variations of depth in the talweg profile not nearly so large (cf. fig. 28 from Öre Älv).

The great variation in width and depth affects the flow conditions and transportation of material in certain respects. This is illustrated in the diagram on the right in fig. 41, showing the area of the river's transverse section for different river levels (140.0, 141.0, 142.0, and 143.0 m at the gage at Stöllet). At low water the cross section at the sharp bends is large compared with that of the intermediate stretches. But as the river stage rises the difference tends to decrease, or even to be reversed. This means that the average flow velocity is relatively low at the bends and relatively high along the stretches between them at low water, whereas at high water the current is rapid at the bends, and river activity there heightened.

This implies a rhythmic movement of bed load: at low water stages material is taken from the intermediate stretches of the river and deposited at the bends, while at high water scouring occurs at the bends and there is a general transportation of sediment. Similar observations have been made in connection with laboratory investigations in America (cf. FRIEDKIN 1945, p. 7).

The course between Öjenäs and Ämtbjörk

The morphology of the river bed in the southern part of the meander zone in Ekshärad parish is different. Fig. 42 is a map of the river between Öjenäs and Ämtbjörk. The soundings on which the map is based were carried out by Uddeholms AB and Klarälvens Flottningsförening. The positions of the sounding sections are marked by straight lines.

The regular meander pattern is absent here. The variations in the width of the river are nevertheless considerable: from 150 m in the north to 350 m in the broad bulge between Öjenäs and Bergsäng. The greatest depth is no more than 5 m, and it does not occur in the narrowest part of the river.

After the relatively straight and fairly deep northern part of the section under consideration the channel widens and at the same time becomes more shallow. The central part of the bulge may best be described as a large horse-shoe shaped sand-bank. The edge of the convex part of this sand-bank along the whole of the western side and part of the eastern is a steep distal slope with a deep channel beyond. The greatest depth occurs in the western channel.

The difference between the morphology of the river bed in this part of the meander zone and in the part in Norra Ny parish already described is wholly attributable to the difference in the transportation and deposition of sediment. In the regularly curved meander loops of Norra Ny sediment is deposited on only one side of the channel, which accounts for the characteristically asymmetric transverse section. But in the present locality in Ekshärad sediment is deposited in the middle of the river, so that a central sand-bank is formed. This sand-bank may be regarded as a delta lobe, though it is not possible for it to develop in the way that a normal delta lobe does. When sediment is

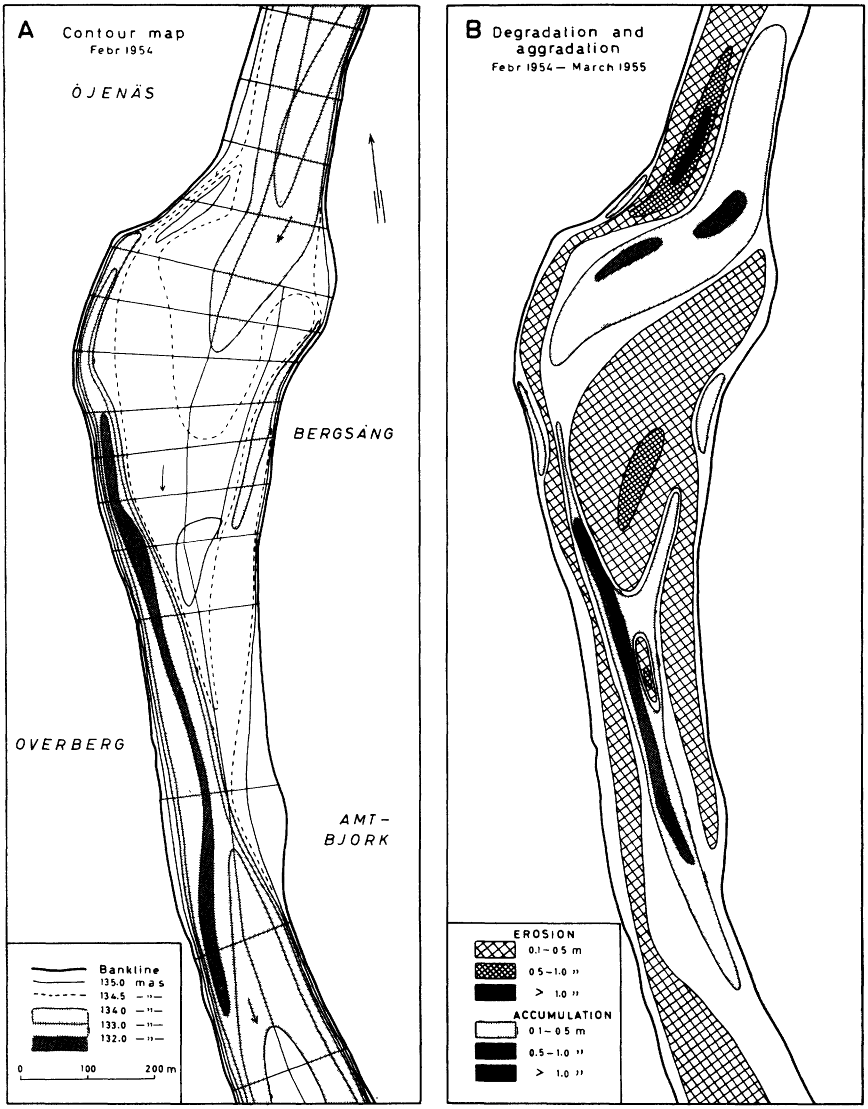

Fig. 42. Map of the river between Öjenäs and Ämtbjörk in Ekshärad parish.

deposited on the lobe the river is diverted outward at *both sides* of the lobe, with a consequent tendency to lateral erosion. The shore line recedes where the erosive force is greatest and the resistance to erosion least. According to Pl. I there has been some lateral erosion on both sides of the bulge in the region of the map.

The above account of how the shore lines and the river bed develop is confirmed by soundings carried out on the same stretch of river a year later. They made it possible to

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follow changes in the bed, and an idea of the transportation of bed load could also be obtained (map on the right in fig. 42).

It was found that some parts of the delta lobe had been degraded and others aggraded during the year. The degradation and aggradation form a fairly regular pattern, with erosion and deposition alternating. The narrow belt of sedimentation near the distal slope is particularly notable. To a great extent the whole pattern has been formed by a downstream movement of sand bars over the surface of the delta lobe. It is probable that if the water discharge were different there might be a tendency to renewed erosion of the channels along the steep distal slopes, provided lateral erosion were retarded by the resistance of the river banks to erosion.

The transverse bars

Under certain wind conditions it may be observed that the water surface exhibits narrow parallel streaks where the waves are more "choppy" than elsewhere (fig. 43). The general direction of these streaks is at right angles to the direction of flow, and the distance between them varies usually between 10 and 20 m. The cause must be variations in the flow, which give rise to the choppy waves through interference with wind-generated waves. When the weather is completely calm it is possible to see that the surface of the water is not flat, but slightly undulated, with the same pattern as that just mentioned.

Both phenomena are associated with the same cause: irregularities in the bed give rise to variations in the rate of flow and small deviations from the normal surface level. The irregularities are the transverse bars mentioned on p. 207.

Transverse bars occur almost throughout the meandering course of Klarälven. They have been observed everywhere except in the deep holes where the river impinges against

Fig. 43. Parallel streaks on the water surface, indicating the position of transverse bars. Looking downstream from Stärnäs (June 1952).

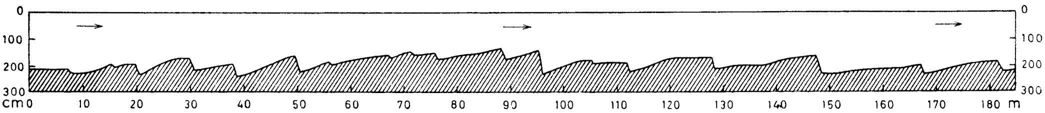

Fig. 44. Longitudinal section of the river bed near Baskenäs, showing transverse bars.

the valley sides, and at places where the bed material is either exceptionally coarse or exceptionally fine, and where the flow is disturbed by some obstacle.

De Geer (p. 76) has given one of the few exact descriptions of this important morphological element. De Geer, however, has only described bars from the part of the river bed that is easiest to observe, that is, the shallows off the downstream sides of meander headlands, which are exposed at low water. In the experience of the author, transverse bars are a normal feature of the bed configuration in rivers where the bottom is sandy and there is relatively large transport of bed load.

As mentioned, the transverse bars are as a rule nearly perpendicular to the direction of flow. But they curve somewhat downstream where the water deepens. Like ripples, they often anastomose. Also, secondary smaller bars may be superposed on the larger bars.

Fig. 44 shows a longitudinal section of the river bed, measured by soundings at intervals of 1 m along the course between Baskenäs and Ändenäs. The direction of flow is from left to right in the figure. The proximal and distal sides of the transverse bars are clearly discernible. The "wavelength" in this part of the river is between 5 and 15 m, and the height of the steeper distal slope between 10 and 70 cm. It is noteworthy that the upstream surface often is somewhat convex.

Fig. 45 is a large-scale map of a sand-bank between Transtrand and Brönäs in Dalby parish (a photograph from the same region is to be found on p. 208). When the survey was

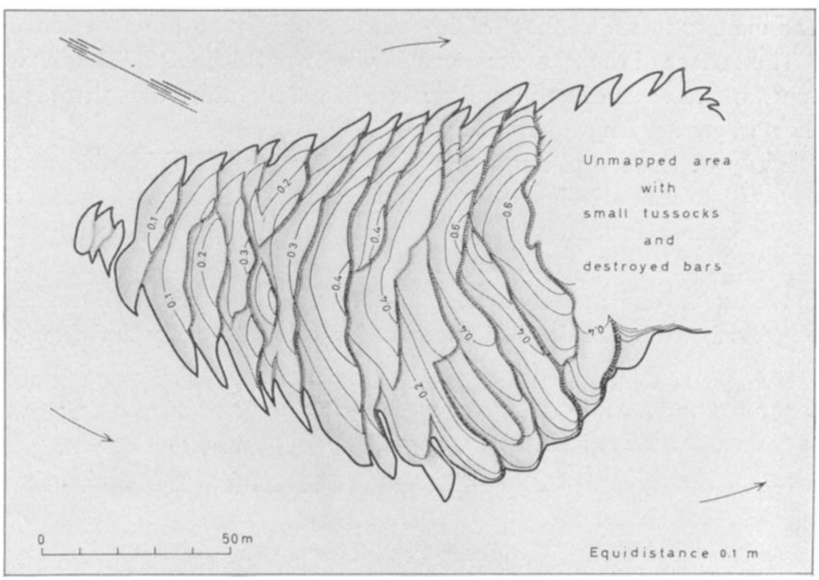

Fig. 45. Detail map of a sand bank with transverse bars, projecting as an islet at low-water. Contours at intervals of 0.1 m. The flow direction is indicated by arrows (July 1953).

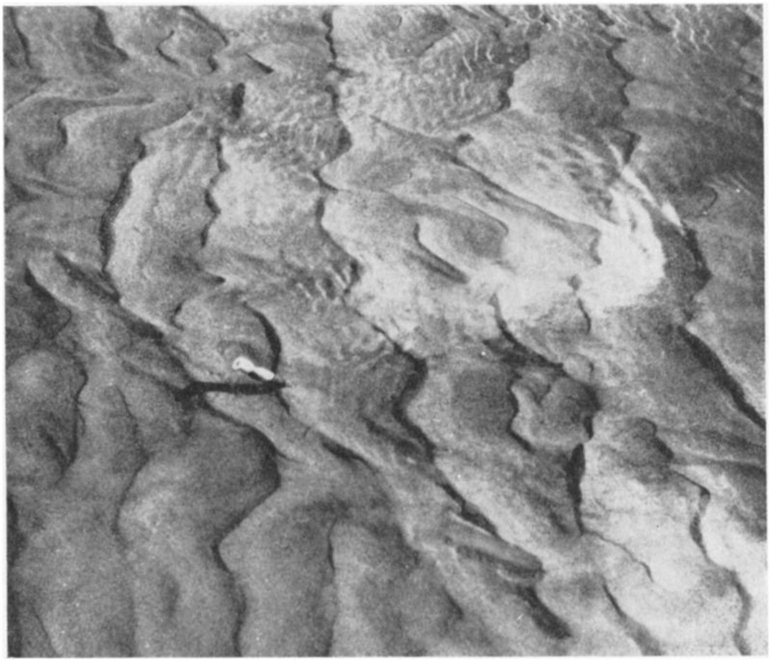

Fig. 46. Ripples on the surface of a transverse bar (from Öre älv, Sept. 1952).

made the river level was falling rather fast, and the bars are therefore well preserved. There is an observable tendency for small bars to develop on the larger bars, for the direction to bend downstream when deeper water is approached, and to anastomosis.

The importance of transverse bars for the transportation of bed load has already been stressed (p. 207), and a hypothetical explanation of how they develop has been put forward. The manner in which they originate is accordingly not discussed in this chapter.

The surface of the transverse bars is often rippled (fig. 46). The ripples are of the normal type, and have so often been described in the literature that there is no need to consider them at greater length here (cf. p. 206).

Study of the morphology of the meander headlands, erosion scarps, and river bed, have led to a relatively thorough understanding of the processes involved in the creation of the various formations of the alluvial valley, from the erosion of erosion scarps, via transportation in ripples and transverse bars, to deposition in the point bars from which the meander headlands are built up. But many important problems still remain, and cannot be solved without a detailed examination of stratigraphy and flow conditions.

STRATIGRAPHY AND COMPOSITION OF SEDIMENTS

The surface of unconformity

Since the present river channel and the recent valley bottom have been formed while the river was cutting into the older fjord sediment, the contact surface between the newer river sediment and the underlying fjord sediment will be a surface of unconformity. This surface of unconformity is still under development and altered by the river channel as downcutting proceeds and the meander system moves down the valley.

With a knowledge of the river bed's morphology, and paying due regard to the manner in which vertical and lateral erosion proceed, it is easy to find the general form of the surface of unconformity. In a *transverse section* it should be lowest near the sides of the valley, where it has been generated by the deep holes in the river bed, and highest in the central part of the valley where it has originated at the bottom of the more shallow parts of the river channel. In a central *valley section* it should be highest at each erosion scarp and then slope downwards under the meander headland until it reaches a lowest point under the present channel on the downstream side of the meander.

De Geer (p. 158) measured the level of the surface of unconformity and found that its position agreed well with what was expected. If we suppose that the overlying sediment were removed, the surface of unconformity would look like a series of convex surfaces, each corresponding to a meander loop, seemingly overlapping the one upstream, and with a slight downstream slope.

A typical stratigraphic section

Since both the newer river sediment and the underlying older fjord sediment are often exposed in the bare erosion scarps, these are excellent places to examine stratigraphy. De Geer (pp. 84 ff, and Tafla 2) has described the sediments in 12 profiles from erosion scarps in the area he investigated. These show the most important features in the structure of the meander headland. But no mechanical analyses of grain size were undertaken, and in addition some important stratigraphic features are neglected—perhaps because no attention was paid to them at the time when De Geer carried out his investigation; accordingly a detailed description may contribute to our knowledge of the processes of formation.

At several localities in the region investigated by De Geer, as well as on other meanders, a detailed study of the stratigraphy has been made, and samples taken for mechanical analysis.[1] The stratigraphic structure at a representative locality may be seen from fig. 47.

The figure shows a section of the erosion scarp at Ljusnästorp, just above the "step" (näsavsatsen) in the terrain, and near sounding section 27 of fig. 41. The point bar formations of the land there are cut through at right angles by the erosion scarp. This means that at the time when the river sediment of the section was laid down *the water moved in a direction at right angles to the plane of the section, in the direction away from the observer.* At the same time *the river channel moved slowly from left to right in the figure* as lateral erosion proceeded. So we may bear in mind that when the sediment was being deposited the erosion scarp of the river was situated to the right, and the side of the river where sedimentation occurred to the left.

[1] Detail investigations have been made of the erosion scarps at Uggenäs, Stärnäs, Brönäs, Krusmon, Baskenäs, Ändenäs, Ljusnästorp, Björby, Stöllet, Gravolsmon, Stensnäs, Götnäs, Ginbergsängen, and Berga.

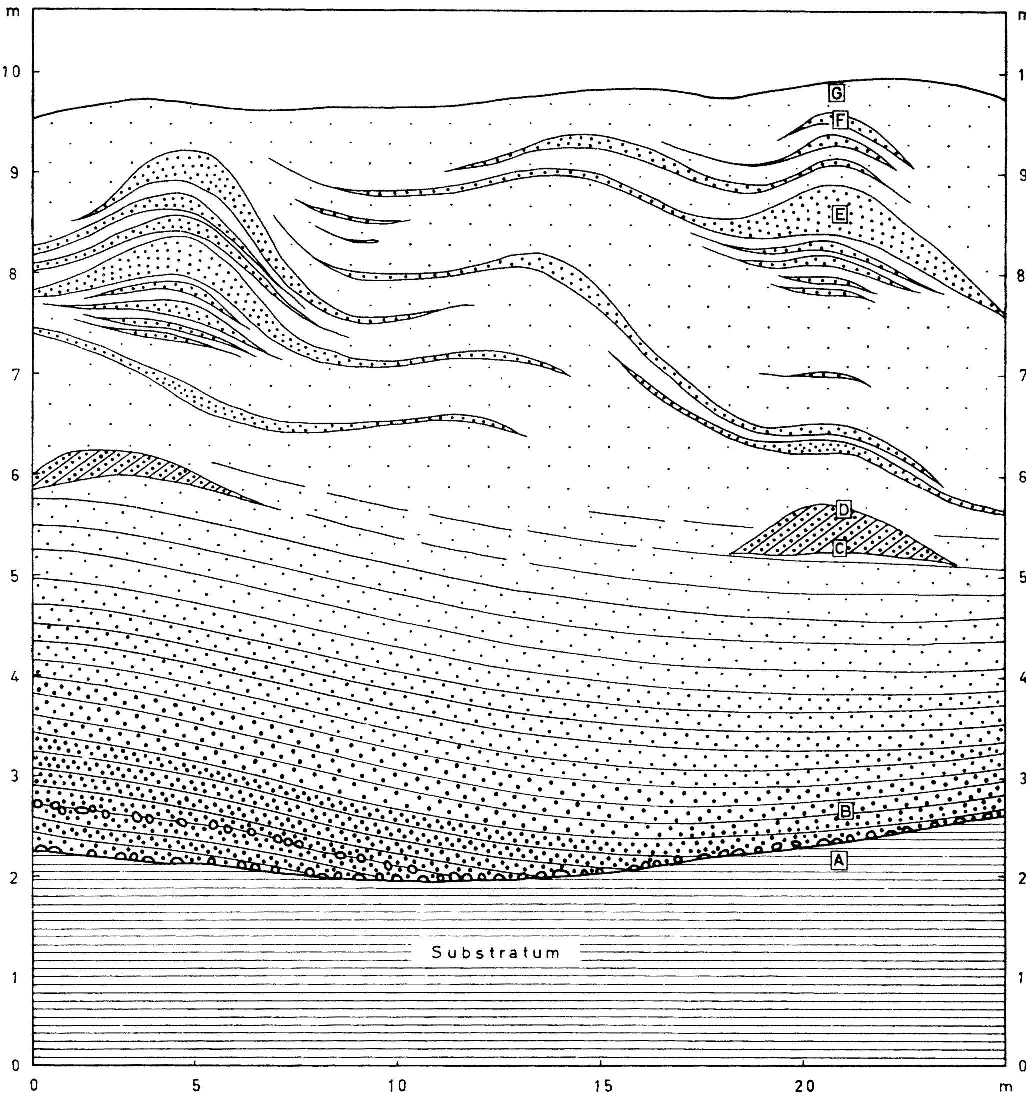

Fig. 47. Stratigraphic section from the erosion scarp at Ljusnästorp. The river deposits are cut through at right angles to the former flow direction by the recent erosion scarp. Explanation in the text.

During the first stage in the evolution the erosion scarp and the river channel moved across the region from left to right. Some of the old fjord sediment was eroded, cutting out the surface of unconformity. The substratum in this locality is varved silt. Its grain size distribution is shown in the cumulative curve A of fig. 48.

The actual surface of unconformity is overlain by a layer of gravel, which is reddish in colour on account of iron oxide. The thickness of the layer is not greater than the diameter of the coarsest particles in it, i.e. c. 30—50 mm. These particles are so coarse that

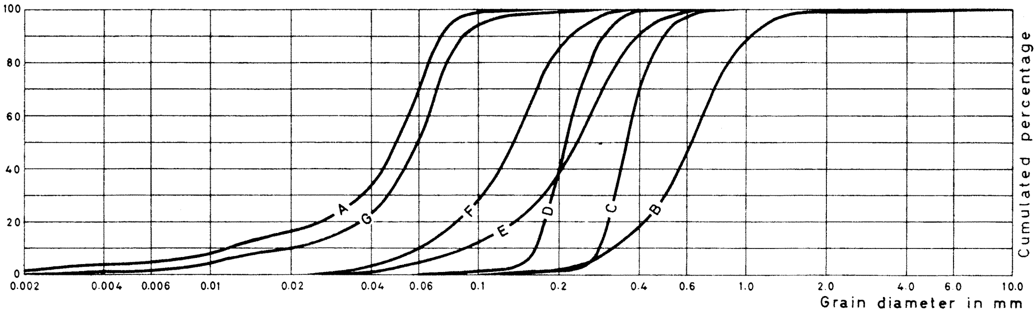

Fig. 48. Cumulative curves for the grain-size distributions of the samples A—G in fig. 47.

they could not be transported with ease, and they therefore accumulated on the river bottom. Similar coarse or even coarser material often occurs on the bottom of the present main river channel. Thin strata of coarse material may even occur in the river sediment above the surface of unconformity; they are to be regarded as due to enrichment of coarse residual material on a temporary surface of erosion.

It is not everywhere that a layer of coarse material occurs immediately above the unconformity between fjord sediment and river sediment, although normally it does so. Occasionally there is not an enrichment of coarser particles in this layer, but instead an enrichment of some heavy mineral, usually magnetite, in a layer some mm thick. Such laminae also occur occasionally higher up in the sequence of strata (cf. p. 199).

Above the surface of unconformity and the layer of gravel there is 2—3 m of material that was mainly transported as bed load (analysis B in fig. 48). If another section is made, preferably at right angles to the section of the figure, and the material in the surface of the section allowed to dry, it may be seen that most of the sediment consists of a large number of strata of cross-laminated material. This material was evidently laid down on distal slopes which moved in the direction of the river's flow. The stratification must have been due to migrating transverse bars. The thickness of the strata varies considerably, and small erosion surfaces or minor unconformities are common. The actual stratification is therefore less regular than it may appear from the schematic treatment in the figure, where this type of sediment is denoted by a dotted surface and relatively parallel lines. In the upper part of this layer the cross-lamination is often absent, and there does not appear to be a sharp boundary against overlying deposits of suspended material.

Roughly in the middle of the section there are two lenses of cross-laminated sediment. The dip of the bedding is not downstream, but to the left, towards the sedimentation side of the river as it was then. Examination of several other sections has revealed that such lenses often occur at the base of point bar deposits. They may be regarded as embryonic point bars. Fig. 49 shows a photograph of a lens of this type from another section on the erosion scarp at Ljusnästorp.

It quite often happens that such a lens does not develop to a point bar, and it may also happen that point bars develop without there being any embryonic form of this kind. The development of point bars will be considered in detail in a later section (p. 288).

The grain-size distribution of the sediment is shown in analyses C and D, where C is from the bottom of the lens and D from near the top. Both samples may be classified as bed-load material, on account of the cross-lamination, and it will be seen from the cumulative curves that they are well sorted.

Above the lenses with cross-lamination the material becomes on the whole increasingly fine-grained, and it is evident that it is a deposit of suspended sediment. But there are some intervening strata of slightly coarser material, which are indicated in the figure by dotted bands. The shape of these layers marks the upper surface of the young part of the meander headland at various stages in its development, that is, successive surfaces of the point bars. It may be noticed that the strata are usually thickest where they are highest, i.e. at the summits of the point bars. Usually they cannot be traced more than some 20 m, although it is certain that they occur as thin laminae of more fine-grained material farther inland on the headland as it was then.

These strata must be interpreted as coherent deposits of suspended material, laid down during short periods when the river was in flood and contained much suspended material. In the majority of cases it must have been a heavy spring flood that left them behind, but one cannot expect to read some sort of "varve chronology" from them, since in all probability the river did not leave any trace in years of moderate spring floods. The grain-size distribution is shown in analyses E and F. E is from a stratum of unusual thick-

Fig. 49. Embryonic point bar in cross section. From the erosion scarp at Ljusnästorp (July 1953).

ness, which was probably formed by an exceptionally heavy flood. Although the material was transported in suspension, the median grain size in E is somewhat larger than that for the bed load material in D (cf. p. 218).

The river channel slowly moved to the right while these deposits were being laid down, and successive embryonic point bars developed. The distance from the more mature point bars to the river channel increased, and only at very high river stages were the point bars inundated, whereupon they received a slight addition of very fine material (analysis G). If the lateral movement of the river stagnated for a time the repeated occasional addition of such material could have finally raised the land surface to the highest level reached by the river. The pronounced undulation of the point-bar relief would then have been flattened out, making the land surface relatively even. This is what has occurred in the region of the section considered here.

It is typical that the land surface above the step formations (näsavsatserna) previously described is often relatively even. This fact supports the explanation advanced for the formation of these steps, that they are a consequence of a stagnation in the lateral movement of the river followed by a resumption of a more rapid lateral movement.

At the top of the section there is a layer some 20 cm thick of cultivated soil containing humus.

Stages in the deposition of the headland sediments

The section described above may be considered as typical of at least the central parts of the valley. It has been found to reflect several stages in the genesis of the sediments of a headland:

1. *The surface of unconformity was cut out by the laterally moving river channel.*

2. *Bed load was deposited by migrating transverse bars.*

3. *Embryonic point bars were laid down on the underlying bed-load deposits.*

4. *The point bars formed through the deposition of suspended material, and their relief was accentuated by successive deposits.*

5. *Fine suspended material was deposited almost up to the highest river level, and the point-bar relief was smoothed out.*

One or more of these stages may have been of minor importance, or even quite absent, depending on the local conditions. Factors exerting an influence on the development are hydrological conditions, the rate of lateral erosion, vegetation, and the meander loops' shape and position with respect to the sides of the valley.

The thicknesses of the two types of deposit, the bed-load material and the suspended, varies widely. The total thickness is greatest near the sides of the valley. Although DE GEER makes no definite distinction between material transported as bed load and suspended material, it seems possible to conclude from the data he gives that in his 12 sections the thickness of the bed-load material varied between 0.9 and 5.8 m, and that of the suspended material between 0.8 and 6.5 m (DE GEER, p. 95). This accords quite well with the author's own observations, where the figures all lie within these limits. However, it should be stressed that it is not possible to give exact measurements except of the total thickness, since the boundary between bed-load material and suspended material is always diffuse.

Some analysis from the present river bed

For comparison with the grain-size analyses of the samples from the erosion scarp at Ljusnästorp some analyses of samples from the present river bed are considered below.

Fig. 50 includes cumulative curves for altogether 16 samples from 4 transverse sections of the river in the southern part of the meander zone, on the stretch between Öjenäs and Edebäck. The samples were taken with a scoop, and were therefore disturbed. The sampling points were evenly distributed over each transverse section.

The grain-size distribution is almost the same for all the samples: they are well-sorted, with a median particle diameter of 0.4—0.6 mm, and very little material finer than 0.2 mm. The most similar analysis among the other samples is B of fig. 48. The grain-size distribution shows that the river bed consists of material which is subjected to active bed-load movement, at least occasionally. Only two samples deviate from the normal type. Sample 5 from the section Bergsäng consists of well-sorted very fine sand and silt. This material is a deposit of suspended material, indicating that there has been no transport of bed load at this point for some time. Sample 2 from the same section, on the other hand, has a secondary maximum at c. 6 mm. This type of distribution indicates that the river bed at the corresponding point is subject to scouring and enrichment of the coarser fractions.

Fig. 51 shows some analyses of samples from the downstream side of the headland Sannersmon in Dalby parish. Sample 1 is from the crest of the point bar nearest the river. Salix vegetation grew at the point where the sample was taken. The sample consists of material transported in suspension (comparable with F of fig. 48).

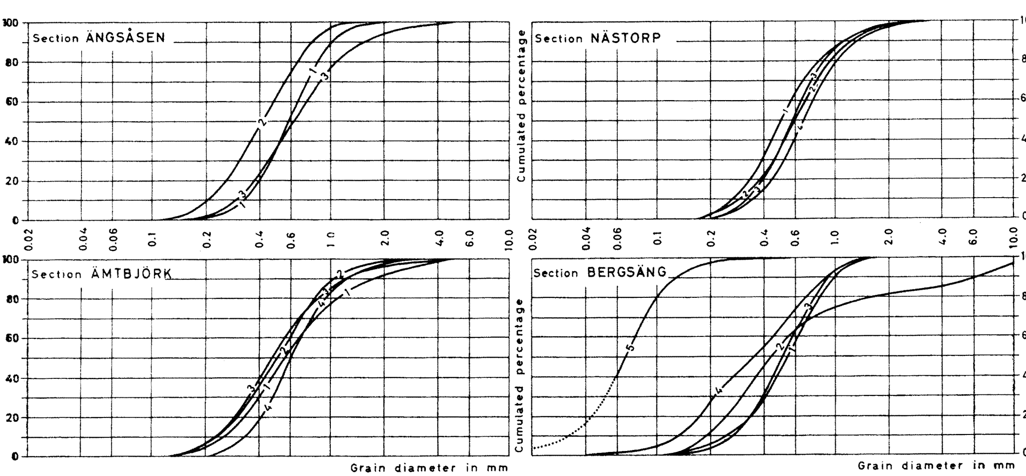

Fig. 50. Cumulative curves for the grain-size distributions of bed samples from the river between Öjenäs and Edebäck.

Samples 2 and 3 are from the same point bar, but farther downstream, where there is as yet no cover of vegetation. It is probable that the material was transported mainly as bed load, and the distribution is similar to that of sample D in fig. 48.

Sample 4 is from the surface of a migrating transverse bar, at a level some 0.5 m below the average river stage but above the water when the sample was taken. The sample was taken half-way between two distal slopes. Finally, sample 5 is from the distal slope of the same transverse bar, half-way between the top and the bottom of the slope. These last two samples most resemble sample B of fig. 48, but are somewhat finer. This is only natural when it is considered that B originated lower down in the sequence of sediment than 4 and 5.

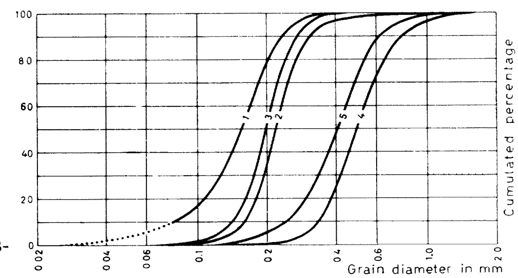

Fig. 51. Cumulative curves for the grain-size distributions of samples from Sannersmon.

An interesting detail concerning the degree of sorting of the various samples may be noted in both fig. 48 and fig. 51. The most well-sorted samples (D, C, 2, and 3) are all on the border between bed load and suspended material. This observation confirms what INMAN (1949, p. 67) found from his investigation of the degree of sorting of sediments in the light of fluid mechanics: "Sediments with median diameters near the grade of fine sand are the best sorted; sediments coarser and finer are more poorly sorted." It may also be noted that sample E with about the same median grain diameter is considerably less well-sorted. This is a suspended sediment.

Some observations on the fjord sediment

Although observations on the fjord sediment are perhaps a little outside the main scope of the present investigation, a few observations may not be quite out of place.

The fjord sediment, which is usually varved, sometimes has cracks and small faults running through it. The erosion surface between the river sediment and the substratum always cuts through these cracks, which proves they are older than the surface of unconformity. Fig. 52 is a sketch of a part of this surface at Baskenäs. The dislocations probably arose during the consolidation of the deep deposits when the fjord was being filled in, and subsequently during the early postglacial period.

The surface of unconformity is often uneven in its details, especially where it cuts through fine-grained varved sediment. The winter laminae, which are the most fine-grained and therefore the most erosion-resistant, stand out as small escarpments. The scarp slope is often ragged. The erosion and entrainment of particles must often have taken place as a breaking away of lumps of material along cracks and bedding planes. Fig. 53 shows such a miniature cuesta landscape. The diagram was drawn from a sketch made at the erosion scarp in Ljusnästorp.

According to the analysis in fig. 48, the fjord sediment at Ljusnästorp consists of very fine sand and silt. But the substratum is not always of this composition. The variations are large, and the material may be either coarser or finer than in the sample considered here. In several places (e.g. at Krusmon) the grain size is on the boundary between bed load and suspended material, and sometimes asymmetric ripples may be observed in the fjord sediment. In such cases it is not possible to trace varves reflecting an annual rhythm of deposition.

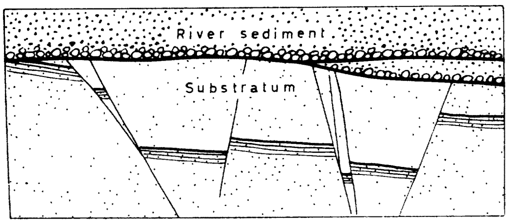

Fig. 52. Sketch of the surface of unconformity between underlying fjord sediment and overlying river sediment at Baskenäs. Note the dislocations in the substratum. The height of the section is c. 15 cm.

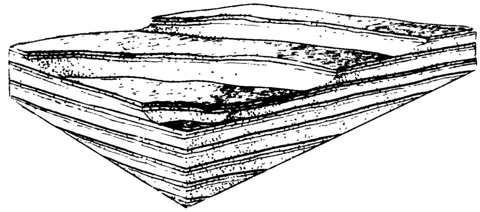

Fig. 53. Sketch of the surface of unconformity at Ljusnästorp, showing the miniature escarpments formed by the erosion-resistant winter laminae. The thickness of the laminae is c. 4 cm.

The borings made by Falk (1953) at Stärnäs, to find the surface of unconformity between the river sediment and the fjord sediment, did not reveal this surface, despite the fact that the borings certainly penetrated into the fjord sediment. This means that there is no appreciable difference between the grain-size distributions of the two sediments at that place. But just south of Stärnäs, on the erosion scarp at Brönäs, the unconformity is quite distinct, and the difference in grain sizes considerable.

Borings undertaken in connection with a projected bridge between Fastnäs and Åstrand in Norra Ny parish reached a level of 118.30 m, i.e. 20 m below the average level of the present river, without finding other material than sand (Statens Geotekniska Institut 1955). But, as already mentioned, the river has elsewhere excavated down to moraine or coarse glaciofluvial material at some places in its meandering course.

The observations mentioned above indicate that sedimentation may have occurred rather irregularly as the ice receded, which means that the rate of retreat of the ice may have varied to some degree. However, the author's material is altogether inadequate for an attempt to reconstruct the course of the process of deglaciation. This problem will be treated by J. Lundqvist in his description accompanying the geological map of Värmland, shortly to be published.

FLOW CONDITIONS AND THEIR SIGNIFICANCE FOR THE PROCESSES OF EROSION, TRANSPORT, AND SEDIMENTATION

Qualitative aspects

The preceding sections have provided a picture of the morphological and stratigraphic consequences of the river's activity. An account has been given of the processes involved in the movement of the river channel and the meander system, and in the genesis of the sedimentary formations which make up the headlands of the valley floor. But a study of the morphology and stratigraphy does not suffice for an understanding of the causes of this evolution. If one wishes to know why the river developed a meandering course in the first place, and subsequently has retained the pattern, and why the river erodes and aggrades its bed in accordance with the regular scheme that has been described, it is necessary to study what is undoubtedly the primary factor in all fluvial processes, namely, the conditions of flow in the river.

Some characteristics of the flow in the meander zone have already been referred to. The average velocity in the deep holes near the valley sides relative to that in the gently curved parts of the meanders was discussed on p. 268. It was pointed out that the velocity in the holes varies greatly from high water to low water, while the difference is much smaller in the wider parts of the meander course. With the help of the diagram on the right in fig. 41 and a computed rating curve giving the discharge as a function of water stage it is possible to calculate the average velocity in each section at different volumes of flow.

There has also been a discussion of phenomena which enable the existence and position of transverse bars to be traced from the appearance of the river surface on certain occasions (p. 270).

Although the average velocity of the water passing a particular transverse section is of great importance as regards the morphological activity of the river, it is of course by no means the only decisive factor. The distribution of flow velocity over the section, the occurrence of secondary flow and separation phenomena, the state of turbulence, and turbulent diffusion all help to create the morphological pattern, and to determine the character and intensity of the processes at work.

Under certain weather conditions, especially when the weather is calm and rainy, it is possible to observe surface phenomena which are associated with the state of turbulence in the river. The photograph in fig. 54 was taken from the erosion scarp at Baskenäs in a direction obliquely downstream. It will be seen that the surface of the water consists of large lighter patches separated by darker strips of irregular shape. Closer study of the manner in which this pattern changes with the motion downstream reveals that it corresponds to turbulent movement of the water. The lighter patches in the photograph are regions where the water is slowly welling up to the surface, while the darker ones are regions where the water moves downward from the surface. Expressed in other words, the pattern reflects patches of divergent and convergent motion. In the photograph

Fig. 54. Surface pattern of lighter patches and darker strips, showing the turbulent movement of the water (Baskenäs Aug. 1953).

these bodies of turbulence have a size comparable with the depth of the water, 2—4 m.

Besides this feature of the macroturbulence there is another phenomenon traceable in the photograph. About 10 m from the erosion scarp, slightly inland from the line of greatest depth in the main channel, there is an almost unbroken dark band, which runs nearly parallel to the bank. In favourable light the effect is striking. The dark band shows that in this zone there is markedly convergent and downward-directed flow. The phenomenon can hardly be explained otherwise than as an indication of secondary flow of the type discussed on p. 163. The phenomenon is even more distinct on the occasions when there is much pollen floating on the surface of the water. When there is no wind the pollen floats in streaks on the flowing water, and forms a ribbon on the surface a little offshore from the erosion scarp. Even logs and other large floating objects often follow the same course, but their movement is usually dependent on the wind and other factors to a great extent.

Flow velocity in a transverse section

We now pass on from these qualitative aspects to some measurements. A profile of the flow velocity is reproduced in fig. 55. The measurements were made between Baskenäs and Ändenäs when the river was slightly below its mean level (140.88 m at Stöllet). The position of the profile is indicated at the bottom of the figure.

The geometry of the transverse section is typical of a normal meander loop. It is markedly asymmetric: near the erosion scarp the slope is steep, though less than the angle of repose, being c. 20° (cf. p. 202), while towards the other side the river bed rises very slowly. The convexity of the bottom near the sedimentation side is normal, as is the steeper slope immediately adjacent to the point bar, which is covered with vegetation.

The measurements were carried out with an Ott-Kempten current meter suspended by a cable. The distance between successive positions was 10 m, except close to the erosion scarp, where it was 5 m. The flow velocity was determined for depths at intervals of 0.5 m.

The maximum velocity occurs where the water is deepest. The actual maximum for a vertical distribution occurs as a rule just below the surface, but the difference is so small

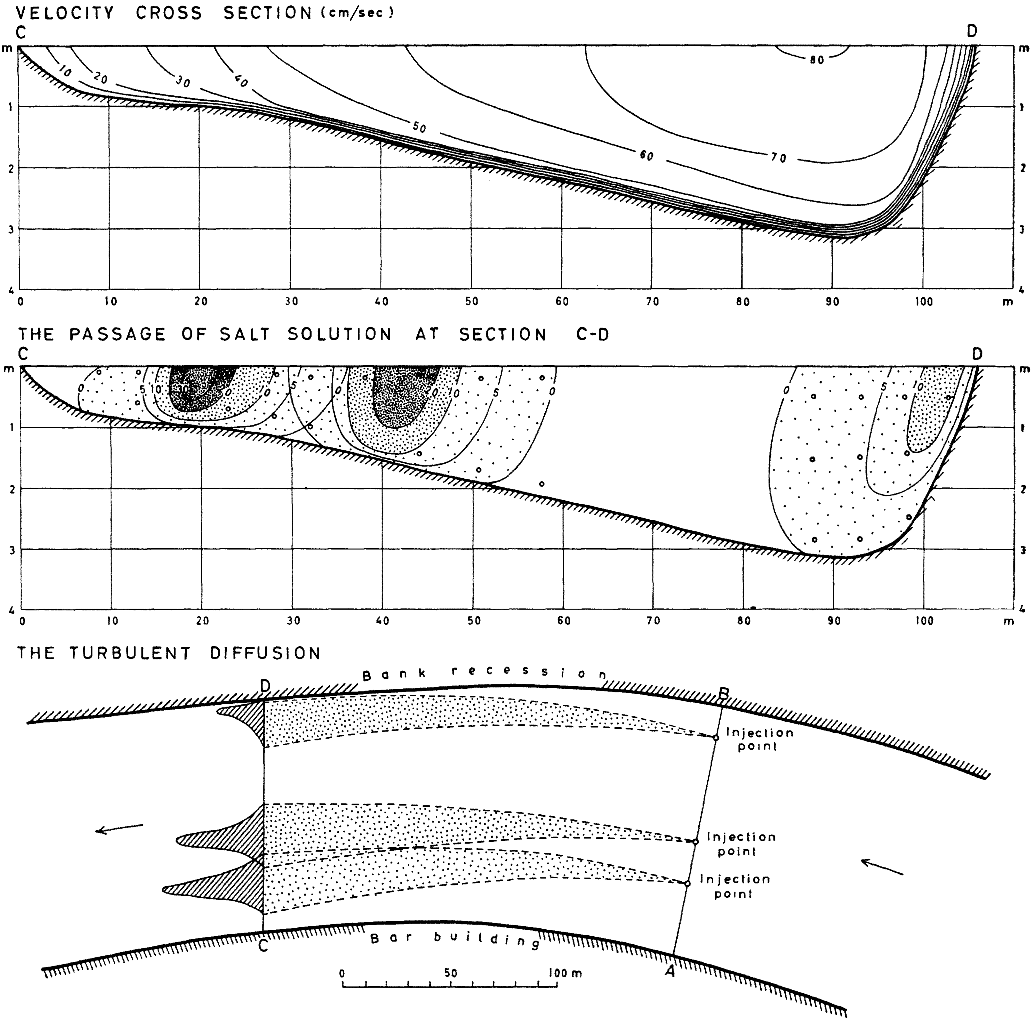

Fig. 55. Flow velocity in a transverse section between Baskenäs and Ändenäs and salt-injection measurements at the same locality. The figures in the middle sub-figure show the relative amounts of salt solution passing each unit cross-sectional area. Further explanation in the text.

that it cannot be shown in the figure. The rapid decrease of the velocity towards the river bed cannot be properly shown on the scale of the figure[1].

Several velocity measurements have been made on the same section on different occasions. As expected, it has been found that the velocities vary with the water discharge, but the distribution over the section remains relatively unchanged. An example of the variation of the velocity with the discharge is that the maximum velocity was 47 cm/sec. when the volume of flow was 45 m^3/sec. (13th Feb. 1954), 81 cm/sec. at 110 m^3/sec. (13th Aug. 1953), and 116 cm/sec. at 310 m^3/sec. (20th July 1953).

Measurements of the flow velocity have also been carried out on several other sections along the meandering course (at Uggenäs in Dalby parish, between Ändenäs and Ljusnästorp, and between Björby and Stöllet in Norra Ny parish). Some local differences in the distribution of velocities occur, but the main pattern was the same for all sections. It is therefore representative to discuss the morphological consequences of the particular distribution of measured flow velocities in fig. 55.

Morphological activity in a transverse section

Application of the results on erosion and the transport and sedimentation of suspended material and bed load arrived at in Chap. II (see in particular fig. 23) allows the transverse section to be divided into three parts with respect to the character of the morphological activity.

1. *From 0 to c. 20 m from the sedimentation side of the river.* The velocity is too small for erosion and transportation of the material of the river bed. The bed material therefore remains stationary. But suspended material can be transported and deposited. The result is that in this part of the transverse section suspended material may be deposited on the bottom. It is also easy to confirm that this does actually happen; for, if the volume of flow remains constant for a time the river bottom near the sedimentation side becomes covered with a thin layer of fine sediment and organic material.

2. *From c. 20 to c. 90 m.* Throughout this part of the transverse section the velocity is sufficient for the entrainment and transportation of bed load. Calculations show that the amount of bed load transported increases towards the main channel. As regards suspended material, there is a continual exchange between the river bed and the layer of water nearest the bottom, so that particles from the bottom are picked up and carried off in suspension, while other particles settle out from the flowing water and are deposited (cf. p. 220). Since the rate at which material is picked up depends on the flow velocity and the state of turbulence, while the rate of deposition mainly depends on the silt content of the water, it may be concluded that deposition of suspended material dominates near

[1] However, it is evident from the calculated average of the equivalent sand roughness, which is 5.2 cm. The calculated values for the vertical ordinates where measurements were made vary between 0.6 and 30 cm. The large variations are mainly due to different positions of the transverse bars with respect to the transverse section of the measurements. Measurements on a vertical ordinate downstream from the distal slope of a bar obviously lead to a higher value of the equivalent sand roughness than measurements upstream from such a slope.

the sedimentation side of the river, whereas the taking up of material into suspension ought to be equally important or even predominant near the main channel.

The morphological process in this region may be described as a transportation of bed load in transverse bars and ripples, with some deposition of fine suspended material nearest the sedimentation side. In Klarälven the water is usually so clear that a transition from a stationary state of the bed to slight movement or to a general movement is directly visible. In the transition region the transverse bars are often not perpendicular to the direction of flow. The crest has a tendency to run obliquely downstream from the sedimentation side in towards the main channel.

3. *From about 90 m to the erosion scarp.* The velocity is sufficient for scouring. The slope of the bed is determined by the velocity distribution and the angle of repose for the material of the bed, and on any cohesion there may be between the particles. In frictional material the slope is never quite as large as the angle of repose (cf. p. 202). Coarser material set in motion on the slope moves obliquely down the slope towards the bottom of the channel, while fine material goes into suspension and is spread by the flowing water (cf. fig. 57). The net result is a removal of material. In the channel itself the situation fluctuates around equilibrium between the deposited and entrained material.

When the volume of flow increases or decreases the boundaries between the three sections are displaced. A decrease means that the velocity also diminishes, and the part of the bed where transport of bed load occurs becomes smaller. The erosion at the erosion scarp decreases in intensity, and may even cease altogether. At very low river stages (corresponding in the section under consideration to c. 50 m^3/sec.), all *general* movement of bed material ceases, and only a certain deposition of suspended material occurs. But even at low water the *local* flow conditions may be such that scouring and transport are maintained. Since the morphology of the river bed is not adapted to the lowest volume of flow it may even happen that the activity in some places becomes greater than at high water.

At high water the erosion at the erosion scarp is intensified, and bed load is transported over almost the whole section. The region where there is only deposition of suspended material becomes smaller, but close to the shallow sedimentation side the deposition of suspended material continues to dominate over the taking-up of material, so that there is still a net deposition of material in this region.

The fluctuations in the water discharge are thus responsible for the interbedding between bed load and suspended material in the river deposits (cf. the stratigraphic section).

It follows from the above that asymmetry of the velocity distribution in a transverse section of the river is in itself sufficient to influence the morphological processes, so that the geometry of the section is changed by erosion and sedimentation. It therefore entails a lateral displacement of the main channel. This process is in actual fact the primary reason for meandering. However, many other factors also exert an influence, and a treatment of the whole meander problem solely on the basis of this fact would be inadequate (p. 291).

Salt-injection measurements

The preceding dicussion may be said to be to some extent a discussion of a static situation. The effect of the flow conditions on the morphological processes has been considered for various situations, but the basis of the discussion has been a single, though typical, transverse section of the river. It is consequently not yet clear to what extent the morphological processes are determined by the differences within a typical transverse section, nor how transportation and sedimentation appear in a three-dimensional picture. For the transportation of bed load the direction of transportation, i.e. the direction of the flow near the bottom is of decisive importance, and for the transportation and deposition of suspended material the direction of flow, the possible occurrence of secondary flow, and, most important, turbulent diffusion.

Direction of flow, secondary flow, and turbulent diffusion have been investigated by *the salt-dilution method* described in Chap. III. Salt solution was injected at different points and at different levels in a particular transverse section, and the spreading of the injected solution measured in other sections at various distances from the first in the downstream direction. Such measurements were carried out at different water stages. The theory and practical applications of the method are dealt with in Chap. III.

The middle and lower sub-figures in fig. 55 show the results of a series of salt-injection measurements in the region where the measurements of flow velocity previously referred to were carried out. Salt solution was injected at three points in the section A—B (see the lower figure). Measurements were made in the section C—D, approximately 200 m downstream. The passage of the "salt wave" was registered at various depths on vertical ordinates which were usually 5 m apart. Exactly the same amount of salt was injected for each measurement. Curves showing the change in the salt concentration with respect to time were constructed for each measurement and each point in accordance with the method indicated in fig. 26.

The transverse section in the middle part of fig. 55 shows curves for the relative amounts of salt solution which passed each unit cross-sectional area. These figures were obtained by graphical integration of the concentration curves, whereupon the relative value obtained was multiplied by the flow velocity at the point in question. The essential point is that *the curves in the transverse section indicate the relative probability that water from the point of injection in A—B reaches various points in C—D.*

The width of the region through which all the injected salt solution passed was 20—30 m. This corresponds to an angle of lateral spread equal to 6—10°, which perhaps may be regarded as unexpectedly small. It may be observed that the lateral spread is somewhat greater near the shallow side of the river than near the deeper side. It is also noteworthy that the curves through points where the same amount of salt passed are as a rule shifted somewhat to the right near the river surface.

The spread of the salt solution in a horizontal plane is shown in the lower figure. The shaded surfaces on section C—D indicate the relative amount of salt passing unit area at the river surface.

The following conclusions may be drawn directly from the figures.

Secondary flow, with surface water moving towards the erosion scarp and bottom water moving towards the other side, is not particularly marked. If it had been more pronounced the lateral turbulent diffusion would have been much greater. But nevertheless there is a discernible secondary flow, which manifests itself in the figure by the displacement of the curves near the river surface towards the right. The divergence between the surface flow and bottom flow appears to be about 1—3°.

The lateral turbulent diffusion is so slight that it takes several kilometres for injected salt solution to become completely mixed with all the water of a transverse section (cf. also p. 230).

Nevertheless, the lateral exchange is of very great significance for the morphological processes. Lateral erosion at the erosion scarp (see fig. 57) and deposition of suspended material near the other side give rise to a lateral gradient in the content of sediment, which, with the help of lateral exchange, leads to a transport of sediment from the erosion side to the sedimentation side in a meander bend. The effect is heightened to some extent by the slight tendency of the water near the bottom, with its greater sediment content, to move towards the sedimentation side, and for the water containing less suspended material to move towards the erosion scarp.

As mentioned, similar measurements were undertaken at different volumes of flow. At very low water stage (volume of flow 45 m^3/sec.) and when there was ice on the river, there was no observable secondary flow. The angle of spread was then not greater than 6°. Unfortunately it has not been possible to carry out measurements at extremely high water discharge, since it has never occurred in the years during which the investigations have been in progress.

Calculation of the amount of deposited suspended sediment

The mathematical treatment of the complex three-dimensional turbulent diffusion process with accompanying secondary flow, gain of material by erosion, and loss of material by deposition, presents very great difficulties, and no attempt has been made to solve the problem. However, a simple calculation may be adduced as a semi-qualitative illustration of the manner in which a lateral gradient in the concentration of suspended material may be supposed to arise.

We assume that the water which passes the two injection points nearest A in the section A—B contains the same average amounts of sediment of various grain sizes in each case, and that the vertical distribution of sediment concentration is in temporary equilibrium. How much of the material has been deposited by the time the water reaches C—D? We will not take into account the lateral exchange, but will treat the problem two-dimensionally, with direct application of equation (59).

Calculation in accordance with (59) of the amount of material in various fractions deposited under the actual flow conditions leads to the values presented in table 3.

Table 3. Residual sediment content in the water at section C—D in percent of the content at section A—B.

Position	Grain size (mm)			
	0.002	0.006	0.02	0.06
Close to the shallow side......	99,84	98.30	82.70	3.02
20 m nearer the main channel.	99.95	99.40	93.54	33.90

The figures clearly show that sedimentation is more rapid nearer the shallow side, and how this gives rise to a marked lateral concentration gradient, especially for the coarser fractions. The actual differences in concentration would be greater on account of the more rapid take-up of material nearer the main channel.

The process must attain a state of equilibrium where the more rapid sedimentation near the shallow side is partially compensated by the supply of further material from the main channel via lateral exchange. The greater the differences in the flow conditions in adjoining parts of a transverse section of the river, the greater the lateral gradient of the concentration of sediment will be, and the more rapidly suspended material will move towards the sedimentation side. This is one of the reasons for the rapid growth of point bars at high water (cf. p. 290).

The flow and diffusion processes discussed above and their consequences for erosion and sedimentation, are of fundamental importance for a river's evolution and morphology. The morphological consequences will be treated at greater length in the following section.

Before leaving this account of flow conditions it should be pointed out that the preceding treatment applies only to the parts of a meander loop where the curvature is even. Conditions in the sharp bends where the river channel turns at the valley side are much more irregular. Since such bends are mainly of local interest, being of less importance for the general development of a meandering river, they have not been considered more closely. The rhythm of flow corresponding to the alternation of high and low water has already been mentioned (p. 268). Another observation worth mentioning is that salt-dilution measurements have shown the turbulent exchange in the sharp bends to be much more intensive than in the smoother parts of the river course, especially when the volume of flow is high. The main cause is the strong macroturbulence, with eddies, rollers, and secondary flow. However, the conditions are irregular, and are dependent on the local morphology.

SOME MORPHOLOGICAL PROBLEMS

The evolution of point bars

The deposition of bed load and suspended sediment on the inner sides of meander bends has been mentioned several times in the preceding sections. The final morphological result of this sedimentation, the formation of the point bars, will now be discussed.

Near the sedimentation side of the river the transportation of bed load occurs mainly as a migration of transverse bars. These generally curve downstream towards the main channel, probably on account of the greater flow velocity as the main channel is approached (cf. fig. 45). The water close to the bottom has a slight tendency to move towards the sedimentation side. The deposition of material on the distal sides of the transverse bars leads to an extension of the bars in towards the sedimentation side. One may say that *the bars migrate downstream, with a weak component of motion towards the sedimentation side of the river.*

During this process the transverse bars often build up a longitudinal bar near the shore (fig. 56). The longitudinal bar starts at the tip of the meander headland, and gradually grows downstream. This growth is mainly due to the material supplied by the transverse bars that move up on to the longitudinal bar. Suspended material also contributes to some extent.

At this stage of its development the longitudinal bar often has a steep "distal" side facing the adjoining shore, and a gentle slope out towards the main channel (fig. 56).

Fig. 56. A point bar at an early stage of development. Notice the steeper side to the left facing the adjoining shore and the more gentle slope to the right out towards the main channel. Between the point bar and the shore there is a typical trough. In the background the highest terrace is visible and also a lower terrace (Baskenäs 1953).

It may seem strange that the movement of the transverse bars does not continue right in to the shore, and that there is in fact often a narrow trough close to the shore. The reason appears to be that the movement of bed load across the longitudinal bar towards the shore, and the deposition on the "distal" side of the bar, cease when the bar has approached so close to the shore that the flow of water from the main river channel towards the narrow trough diminishes or ceases altogether. This in its turn is determined by the relative level of the water in the main channel and in the trough, as well as by the cross section of the trough at the particular river stage.

Such a longitudinal bar is the first stage in the development of a *point bar*. In the stratigraphic profile of fig. 47 it corresponds to the lenticular formation from which the samples C and D were taken. The characteristic feature is that *the steep side faces the adjoining sedimentation shore of the headland.*

As soon as the growing point bar has grown up above the average river level vegetation begins to find a root-hold there. Various Salix species are prominent colonisers, and soon form a thick belt of bushes along the crest of the bar. The point bar now rises rapidly, owing to the deposition of suspended material at high water among the thick vegetation. Deposition of suspended material is greatly encouraged by the vegetation, which creates sufficiently calm water for sedimentation, and a very different flow velocity from that in the adjoining free water. Suspended material is "sucked" into the belt of bushes in consequence of the turbulent diffusion.

The point bar now assumes more the character of *a normal levee. Its steepest side is often that facing the main channel, and the shoreward slope often more gradual;* this is particularly true of the parts of the point bars nearest the valley sides, where the river begins to swing in the other direction, i.e. where the meander reaches its point of inflection. This stage of the point bar's development is represented in the stratigraphic profile by the sample E. The difference between the levels of crest and trough is then greatest, and the relief is often quite pronounced.

As soon as another point bar has begun to develop (usually after 10—20 years), and cut off the supply of coarse-grained suspended material to its predecessor at high water, suspended material is deposited more evenly on both crest and trough of the latter. However, sediment is deposited on the crest of the point bar only at high water. Consequently there is a gradual *smoothing out of the relief* owing to deposition of sediment in the trough. This is the final phase in the evolution of a point bar.

Differences in the surficial morphology of meander headlands may be attributed to the dominance of one or other of the phases of formation described above. Local variations do occur, and the general description does not apply everywhere in detail.

It is important to note that according to this interpretation the formation of point bars is not associated with exceptionally high spring floods or any other catastrophic circumstances, as has sometimes been stated, but is rather a more or less continual process, governed by the normal morphological activity of the meandering river. The rate at which point bars are formed is dependent on the amount of bed load and suspended load transported by the river, the resistance to erosion of the opposite erosion scarp, and of course, the prevailing hydrological conditions.

The development of a meander pattern

The study of morphology and stratigraphy, and of flow conditions in the river, has strengthened the impression that there is an intimate connection between the various manifestations of the river's morphological activity. In general it is not possible to say which of the processes—erosion, transportation, or deposition—is the primary cause of a particular phenomenon.

As already pointed out, an asymmetry in the distribution of flow velocity in some section of a river course implies an asymmetry in the morphological activity, and therefore favours a displacement of the entire river channel.[1] This applies not only to meandering rivers, but also generally, to any irregularity in the course of a river. However, it is only under certain geological and hydrological conditions that the displacement process continues so far and so regularly that meanders develop.

The essential features of the development of a meander loop may be briefly described as follows. The asymmetric distribution of flow velocity over a transverse section of the river creates a tendency to erosion on the outside of a bend and deposition on the inside. This tendency is accentuated by the occurrence of secondary, helicoidal flow phenomena. Lateral turbulent diffusion leads to a certain movement of suspended material towards the sedimentation side. Deposition of bed load and suspended material on the inside of a bend tends to diminish the cross section of the river, so that the average rate of flow tends to increase, and with it the erosive forces acting on the erosion scarp.

If the erosion scarp is stabilised temporarily, e.g. by reinforcement of the bank, the lateral displacement of the river is hindered, while sedimentation on the other side *at first* continues. The average flow velocity accordingly rises, and the asymmetry of the transverse section decreases, until there is no further sedimentation, and all material arriving from upstream continues on downstream. The resulting river channel is narrow and deep. On the other hand, if the material in the erosion scarp is easily erodible, erosion there may proceed more rapidly than sedimentation on the other side. In this case the resulting transverse section is wide and shallow, with the main channel much nearer the erosion side.

It is known that the width of a free meander belt and the curvature of the individual meander bends bear a definite relation to the hydrographic data for the river, especially the volume of flow (cf. HJULSTRÖM 1942, pp. 249 ff, FRIEDKIN 1945, p. 9, and BAULIG 1948, p. 143). It has also been observed that in a freely meandering river the radius of curvature and the width of each loop increase until the radius of curvature attains a maximum (FRIEDKIN 1945, p. 15). The river activity does not cease, however; there continues to be a displacement of the meander loops with roughly unaltered curvature, though more slowly than before. From time to time the cutting-off of a loop or the formation

[1] A similar suggestion concerning the process of displacement of the river channel has been made by LEIGHLY (1932, pp. 9—10), although the foundation for his analysis is not quite the same as the present writer's.

of a chute disturbs the slow but steady movement of the meanders, and a more rapid phase of development is initiated.

It is natural that the widening of a meander loop ceases when the radius of curvature has become so large that a further increase would lead to a cessation of the lateral erosion. *The limiting value of the radius of curvature for a free meander loop is therefore that at which the volume of flow concerned leads to a velocity distribution such that the critical value for erosion of the material in question is just attained at the erosion scarp. The shape of the meander loop thus depends on both the flow conditions and on the material of the river bed.*

The present state of our knowledge is inadequate for a detailed analysis of the connection between the factors involved and the resulting meander pattern. There is no sufficiently detailed and systematically arranged body of empirical facts regarding the development of meanders under different conditions to enable an unobjectionable theory to be arrived at inductively. And at the same time our knowledge of the dynamics of flow and the morphological activity is insufficient to provide a definite solution of the problem deductively. However, it is possible to formulate some questions which require to be answered before the whole complex problem of meanders can be dealt with.

What patterns of flow may be expected in a meander loop of given curvature and transverse section at different volumes of flow? This problem, which must be regarded as the fundamental one, contains several important subsidiary problems, all of which are of importance for the degree of asymmetry in the velocity distribution. There is, for instance, the effect of transverse oscillations in the flowing mass of water (cf. HJULSTRÖM 1942), and the occurrence of secondary flow. For large rivers at relatively high latitudes the question of the Earth's rotation must also be considered. The Coriolis acceleration favours curving to the right in the northern hemisphere, and to the left in the southern.

Another important question is: How does the critical erosion velocity vary with the type of soil and the slope of the eroded surface? This question, which is of great significance as regards lateral erosion, was considered on a theoretical basis in an earlier section (p. 202), but further investigation is desirable.

It is probable that the influence of shore vegetation also modifies the evolution of meanders, and may perhaps give rise to a systematic variation between meander patterns of different climate and vegetation. The question of vegetation's significance is therefore important, though in the present work it is hardly touched upon.

TRANSPORTATION OF SEDIMENT IN KLARÄLVEN

Indirect estimations of the amounts transported

During the postglacial period Klarälven has cut deep into the glaciofluvial sediment in the ancient fjord (p. 244). Large amounts of sediment have been eroded and carried away by the river.

It is difficult, however, to estimate the total amount of material removed from the meander zone between Vingängsjön and Edebäck. The greatest source of error in such an estimate is the uncertainty involved in reconstructing the position of the bottom of the former fjord. If it is assumed that the valley was practically filled with sediment at the beginning of the postglacial period, a rough calculation shows that some $2 \cdot 10^9$ tons of sediment has been removed. This would mean that the mean annual loss of material is of the order of 200,000 tons.

This figure is very uncertain, however, and obviously represents an upper limit. It is difficult to judge how near it may be to the true figure. Erosion and transportation of sediment must have been more pronounced during the first phase of the river's down-cutting, before the threshold of rock at Edsforsen became a local base-level.

Another possible way to estimate the transportation of sediment in recent time is a comparison of the maps and aerial photographs from different dates which were referred to in the description accompanying Pl. 1. Since the parts of the meander headlands eroded away are considerably higher than the parts newly laid down, the movement of the river channel must correspond to a net loss of material.

A computation on the basis of such a comparison is also highly uncertain, mainly because it is difficult to judge how much of the suspended material has been deposited in the regions subject to flooding. It is therefore not worth-while to attempt a detailed calculation. However, a rough estimate may be of interest.

The average loss of land since the beginning of the nineteenth century has been some 13,000 m^2 per year (p. 257). If one assumes that the average difference between the height of the erosion scarp and the new land built up by sedimentation is 4 m, the annual loss of material is about 75,000 tons (the dry weight of the material is taken to be 1.4 tons/m^3 according to a commonly used number).

Water samples

The variation of the suspended sediment content in the vertical direction

In order to obtain a more exact estimate of the present transportation of sediment, during 1953—55 measurements were carried out on the amount of material in suspension, and calculations made on the amount of bed load. The equipment used for taking samples has been described in a preceding section (p. 233).

The content of suspended material varies in a regular manner with the distance from the bottom (p. 214). A single sample from a given vertical section is therefore insufficient, unless one has some other means of determining the vertical distribution of suspended material.

The principles involved in the choosing of sampling points have been discussed in Report No. 8 on "Measurements and Analysis of Sediment Loads in Streams" (1948, p. 38). Five methods are mentioned there for taking samples of suspended material:

a. "A single sample taken at the surface.

b. A single sample taken at 0.6 m depth.

c. Two samples, one taken at the surface, and the other at the bottom, weighted equally.

d. Three samples, taken at surface, mid-depth, and bottom, weighted equally.

e. Three samples, taken at surface, mid-depth, and bottom, with the mid-depth sample given twice the weight of the others."

It is pointed out (p. 39) that "the methods involving three samples are more accurate than the surface and bottom method, and they are sufficiently simple to be handled by unskilled observers". It is also stated (p. 38) that "the second of the three-point methods is preferable".

For the measurements in Klarälven a somewhat modified three-point method was used, for practical reasons. Water samples were generally taken at three different levels in each position, one near the bottom, one 100 cm above the bottom, and one 10 cm below the surface. In computing the mean value, the observations were assigned different weights. These weights were based on the assumption that the vertical distribution of velocity is logarithmic, and the concentration of sediment that given by equation (56). Since the sampling points were at fixed levels, while the depth of water varied, the weights of the different observations varied with the depth of water. The mean was calculated by means of a formula of the type

$$C_M = \frac{k_B\,C_B + k_{100}\,C_{100} + k_S\,C_S}{k_B + k_{100} + k_S} \tag{63}$$

where C_M, C_B, C_{100}, and C_S denote the mean sediment concentration, the concentration at the bottom, at 100 cm above the bottom, and at the surface, respectively. k_B, k_{100}, and k_S are coefficients whose numerical values depend on the depth of water and the velocity distribution.

The above procedure has meant that the calculation of the mean values was more laborious than if the coefficients had been fixed. But it may be assumed that the result is as reliable as can be attained by a three-point sampling method.

Lateral variations in the sediment concentration

The concentration of suspended material varies laterally as well as vertically (cf. p. 288). It is a common observation that there are considerable differences in the content of suspended material in a transverse section (cf. HJULSTRÖM 1935, p. 405, Measurement and Analysis of Sediment Loads in Streams, Report No. 8, 1948, p. 45, COLDWELL 1947, and MITCHELL and CONSTANT 1948).

The author has often observed at high water in the meandering course of Klarälven that erosion gives rise to a strip of highly silt-laden water adjoining the erosion scarp. This strip of water widens downstream on account of turbulent exchange, and the local maximum concentration decreases correspondingly.

The photograph in fig. 57 shows such a turbidity current close to the erosion scarp at Gravolsmon in Norra Ny parish. Under similar conditions water samples were taken at various distances from the erosion scarp at Ljusnästorp. Analysis of the samples showed that the concentration of sediment at the surface 1 m from the erosion scarp was 173 mg/l, 10 m from the shore 35 mg/l, and 50 m from the shore 22 mg/l.

The investigation of turbulent diffusion in the river by means of the salt-dilution method, which has already been described, showed that the internal mixture of flowing water is a

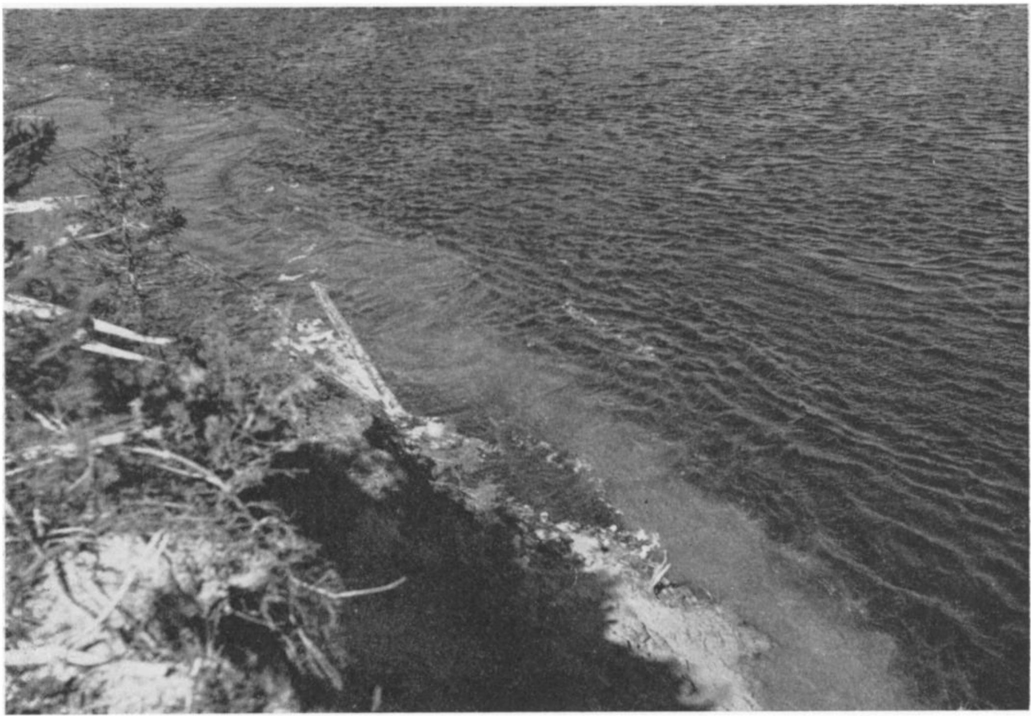

Fig. 57. When erosion is active a distinct strip of silt-laden water is often discernible close to the erosion scarp (Gravolsmon 1955).

relatively slow process, unless the water is rushing along through a narrow channel. Accordingly it cannot be expected that the suspended material from an erosion scarp can become evenly distributed in a river such as Klarälven until the water has flowed some kilometres from the place where the sediment was taken up.

It is therefore necessary to choose between the following alternative ways of taking samples.

1. Sampling at a large number of points in a transverse section, and a calculation of the total load of suspended material. It is then necessary to know the distribution of velocity over the transverse section, and to use it in the calculations.

2. Sampling at a single point or a few points, so chosen that the samples give a representative average value for the entire transverse section. The distribution of velocity need not be known in detail; all that is required is the volume of flow.

The second alternative has been chosen in the present case, mainly to save time and expense. The first alternative would obviously have given a more exact result, but a far greater number of samples would have been required. In applying the second alternative particular care was taken in choosing the sampling points. Flow conditions and erosion were first taken into account before deciding on the sites for sampling.

Sampling sites

During the course of the investigation samples were taken at the following 4 places:

A. *Värnäs in Norra Ny parish.* The sampling site was on a straight part of the river just above the junction with the tributary Våråan. There is no appreciable erosion upstream from the sampling site nearer than the erosion scarp at Ljusnäs. The suspended material is therefore well distributed over the whole transverse section.

B. *Sälje in Ekshärad parish.* This sampling site is also situated on a straight part of the river, and roughly in the middle of the river, about 1 km downstream from the nearest erosion scarp. The depth of water is relatively small for the entire section, and there is an active transportation of bed load in the form of migrating bars whenever the volume of flow is high. The active movement of bed load leads to a high concentration of suspended material in the samples from near the bottom. It is therefore possible to decide from the Sälje samples at what volume of flow the transportation of bed load begins to be significant.

C. *Edebäck in Ekshärad parish.* The sampling site was situated between a small islet in the river, 500 m upstream from Edebäck, and the south-eastern river shore (north of *b* in the name Edebäck on Pl. 1). Erosion along the shore upstream from the sampling site may sometimes lead to disproportionately high concentrations of sediment in the samples, so that the values obtained here must be treated with a certain amount of reservation. Moreover, samples were not taken there during the whole of the observational period.

D. *Edsforsen.* Samples were taken just downstream from the outlet from the power station. The water there is highly turbulent, so that samples from the bottom and from the surface were considered sufficient. The samples from Edsforsen may be taken to provide a value for the total amount of sediment passing the power station, both the material arriving in suspension and that arriving as bed load.

Analysis of the samples

A comprehensive investigation of the denudation of a river basin should include a determination of the amounts of mineral particles, of organic material, and of material in solution. Methods for such an investigation have been treated in detail by HJULSTRÖM 1935 (pp. 389 ff.). The total amount of suspended material is commonly determined by filtering, whereupon the filtered residue is weighed. HJULSTRÖM used the so-called asbestos method for his investigations. The amount of organic material may subsequently be computed by determining the ignition loss. The content of dissolved substances is determined by evaporating the filtrate and weighing the evaporation residue.

It is evident that the choice of method must depend on the purpose of the investigation. The morphological action of the river was the main object of study in the present case. The suspended inorganic matter was therefore the most important component of the sample. With this end in view the method of analysis was simplified as much as possible. The contents of organic material and of substances in solution were not determined.

The water samples were filtered through Munktell's filter paper No. 00 with a diameter of 18.5 cm. The filter paper was then ignited at a temperature of 800° C, and the residue weighed by means of an analysis balance with an accuracy of 0.1 mg. After subtracting the weight of ash from the filter paper the weight of inorganic material suspended in the sample is obtained.

There are two main sources of error in this method. In the first place, the filter paper used is partially permeable for highly dispersed material. Consequently the method cannot be used when the suspended matter is so fine-grained that a considerable proportion of it penetrates the filter. This applies, for instance, to samples from streams or rivers flowing over clay plains: according to HJULSTRÖM's measurements for the drainage basin of Fyrisån, about 20 % of the suspended material there is so fine-grained that it passes through the paper.

In the basin of Klarälven there are no clay plains, and the fjord and river sediments where the suspended material originates contain negligible amounts of clay (cf. previous analyses). It was therefore considered justifiable to use filter paper of the type mentioned. And it was found that in no case was there any detectable turbidity in the filtrate.

The other source of error is the possible breaking down of mineral substances on ignition heating. For instance, carbonates may be transformed to oxides, whereupon the weight of matter is reduced. However, the error from this source must be negligible in the present case, when the composition of the bed rock and soil is considered.

Taking into account the other sources of error, such as weighing errors, material remaining in the sampling container, etc., it seems reasonable to assess *the margin of error* as about 10 % for low sediment contents (less than 10 mg/l), and less for the high sediment contents.

The silt content and the sediment load

The results of the analyses from the four sampling sites are shown in the tables below. The simultaneous value of the discharge at Edsforsen is also stated in the tables, and the calculated total load of suspended inorganic material given in tons per day. It should be observed that the figures for the total load at Värnäs are somewhat uncertain, since this point is 40 kilometres above Edsforsen, and the variations in the volume of flow there are therefore not synchronous with those at Edsforsen. The increase of the drainage area from Värnäs to Edsforsen is 10.5 %. It has been assumed that the volume of flow also increases by 10.5 % from Värnäs to Edsforsen. It is of course not quite correct to assume such proportionality, but here, where there is no necessity for exact accuracy, the results are acceptable.

It is clear from the tables that there is a pronounced positive correlation between the discharge of water and the content of sediment. Really high concentrations of sediment occur only in connection with the spring flood or other high water.

Comparison of the figures for the different localities shows, among other things, that the load of sediment at Värnäs at high water is always considerably lower than at the places farther downstream. This demonstrates clearly that a great deal of the suspended material is taken up by the river on erosion of its bed along the meandering part of its course.

Closer consideration of the relation between the volume of flow and the content of sediment shows that it is not single-valued. The diagram in fig. 58, which shows the sediment content at Edsforsen during the spring flood of 1954, illustrates this.

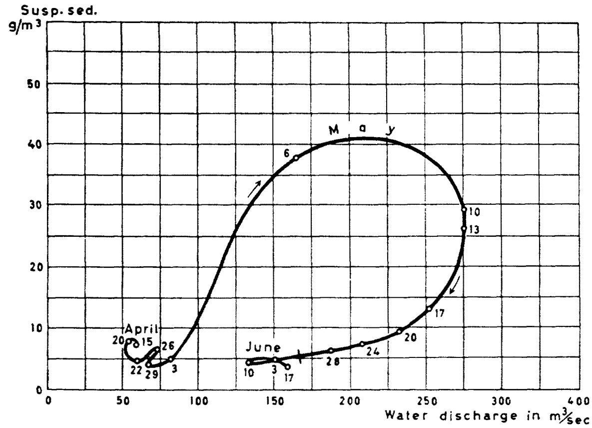

Fig. 58. The sediment content at Edsforsen during the spring flood of 1954.

Table 4. The content of suspended inorganic matter at Värnäs.

Date	Discharge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours
1953 Sept. 3	168	7.0	4.9	4.1	5.3	69
12	136	3.5	3.7	2.7	3.4	36
19	107	4.8	3.8	3.8	4.3	36
26	141	5.6	5.1	3.8	5.0	55
Oct. 3	195	4.4	3.9	3.5	3.9	60
10	129	5.8	4.8	4.6	5.2	52
17	110	3.7	3.3	3.1	3.5	30
24	95	3.4	3.0	3.2	3.2	24
31	107	6.1	4.9	5.0	5.5	46
Nov. 7	177	10.4	9.1	8.5	9.3	128
14	156	4.4	4.3	3.8	4.2	51
28	96	25.0	22.2	17.5	22.7	170
Dec. 12	109	4.8	4.0	3.8	4.4	38
26	85	4.5	—	3.4	4.0	27
1954 Jan. 9	40	5.3	—	3.5	4.4	14
23	50	3.8	—	3.0	3.4	13
Feb. 6	45	(40.3)	—	4.8	4.8	17
20	40	5.7	—	2.9	4.3	13
March 6	42	2.8	—	2.5	2.7	9
20	44	3.1	—	2.8	3.0	10
April 3	42	2.2	—	2.5	2.3	8
15	62	4.2	—	4.8	4.5	22
20	58	8.8	—	7.7	8.3	38

Date	Discharge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours
22	66	6.9	—	7.2	7.0	36
26	68	7.8	7.3	7.1	7.5	40
29	76	8.7	9.6	7.2	8.7	52
May 3	83	8.9	8.8	8.1	8.7	57
6	199	28.2	23.7	22.1	24.0	375
10	295	(38.8)	14.8	12.8	18.6	431
13	279	18.6	17.1	14.1	15.9	348
17	255	12.5	(5.0)	9.0	10.1	203
20	224	6.8	6.1	6.3	6.3	111
24	209	8.7	6.8	6.8	7.2	118
28	186	5.9	5.9	5.8	5.9	86
June 3	143	14.1	8.5	5.1	9.2	106
10	157	16.9	6.7	7.7	10.1	124
17	152	4.5	4.6	3.9	4.4	53
24	141	5.2	4.2	3.9	4.5	50
July 1	158	5.1	4.2	4.0	4.5	56
8	97	3.9	3.1	3.9	3.6	27
15	110	5.5	4.2	6.1	5.2	45
22	143	5.3	4.6	4.6	4.9	55
29	145	7.5	7.1	6.7	7.1	81
Aug. 5	152	5.1	4.3	2.9	4.1	49
12	241	7.2	6.5	5.9	6.4	121
19	177	5.1	3.7	3.2	4.0	56

Table 5. The content of suspended inorganic matter at Sälje.

Date	Dis-charge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours	Date	Dis-charge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours
1954 Apr. 17	72.0	11.3	8.7	8.3	10.5	65	1955 May 2	97.0	11.9	10.1[1]	8.4	11.2	94
20	55.2	7.5	7.8	6.6	7.5	36	5	157.9	31.7	27.0	22.4	29.8	407
22	59.7	10.2	5.9	5.9	8.9	46	9	195.5	58.9	33.9	33.0	46.2	780
26	72.9	8.2	8.1	7.5	8.1	51	12	276.0	86.9	45.9	29.4	59.0	1407
29	66.3	5.3	5.7	4.6	5.3	30	16	190.0	70.8	19.0	13.8	43.9	721
May 3	83.2	6.1	5.4	4.7	5.8	42	19	204.0	62.0	29.0	24.1	44.5	786
6	163.5	67.3	45.6	38.1	55.0	777	23	372.0	75.7	27.9	17.9	39.2	1260
10	276.2	45.8	35.0	29.4	38.2	912	26	220.8	40.5	9.8	9.7	25.1	479
13	276.3	128.2	40.7	21.8	74.9	1788	30	253.0	27.6	19.3	14.8	21.7	474
17	251.5	20.3	17.8	13.2	17.9	389	June 2	269.0	28.1	23.3	13.8	23.3	542
20	232.7	22.0	12.7	14.5	17.7	356	6	280.0	37.8	27.2	14.1	28.8	697
24	206.9	22.1	7.5	6.7	14.6	261	9	295.0	35.8	31.0	14.8	29.7	757
28	185.6	29.6	8.4	6.6	18.6	298	13	217.1	79.1	8.4	5.4	43.2	810
June 3	150.1	9.2	8.3	6.5	8.8	114	16	195.5	11.0	7.4	5.4	8.8	149
10	133.2	7.7	5.3	7.2	7.2	83	20	187.0	10.9	6.8	5.2	8.5	137
17	159.5	15.5	4.9	4.9	12.3	170	23	195.0	7.1	7.0	5.4	6.7	113
24	122.0	20.6	11.2	9.2	15.5	163	27	168.4	8.3	6.3	5.3	7.1	103
July 1	156.9	5.0	4.5	4.0	4.8	65	30	162.3	9.7	6.0	4.6	7.6	107
8	94.5	4.4	4.1	4.8	4.4	36	July 4	150.6	9.3	4.9	5.5	8.0	104
15	95.6	6.3	5.3[1]	5.1	6.0	50	7	179.8	13.1	6.2	5.8	9.6	149
22	160.1	6.2	5.4	5.0	5.7	79	11	135.0	14.2	5.5	6.1	11.7	137
29	123.2	4.9	4.5	3.8	4.7	50	14	139.0	5.3	6.0[1]	4.9	5.4	65
Aug. 5	165.5	7.1	6.6	5.6	6.7	96	18	109.3	14.6	6.2[1]	5.0	12.0	113
12	212.8	25.5	11.5	8.9	18.0	331	21	90.2	4.7	4.8[1]	3.8	4.6	36
19	198.3	10.2	5.8	5.7	8.0	137	Aug. 4	63.0	7.0	4.8[1]	4.5	6.3	34
26	155.0	5.9	5.2	4.4	5.6	75	8	58.0	3.4	4.4[1]	3.7	3.6	18
Sept. 2	124.3	5.3	4.6	4.6	5.1	55	11	68.0	5.4	3.5[1]	4.0	4.9	29
9	96.8	3.4	3.2	3.1	3.3	28	15	60.0	3.9	4.8[1]	3.0	4.0	21
16	134.1	6.4	4.9	7.1	6.2	72							

[1] 60 cm above the river bed.

Table 6. The content of suspended inorganic matter at Edebäck.

Date	Dis-charge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours	Date	Dis-charge m³/sec	Content of silt C_B	C_{100}	C_S	C_M	Load tons/24 hours
1954 Apr. 17	72.0	10.8	9.1	6.9	8.1	50	17	159.5	4.7	4.7	4.1	4.3	59
20	55.2	5.5	6.0	5.2	5.4	26	24	122.0	7.3	4.8	4.6	5.2	54
22	59.7	4.9	4.5	4.7	4.7	24	July 1	156.9	4.9	4.2	7.1	6.1	83
26	72.9	7.3	7.2	7.0	7.1	45	8	94.5	5.0	5.0	2.9	3.7	30
29	66.3	4.2	6.4	4.3	4.7	31	15	95.6	5.7	4.4	5.1	5.1	42
May 3	83.2	4.0	4.3	3.9	4.0	29	22	160.1	6.1	4.7	4.6	4.9	68
6	163.5	44.2	36.7	38.8	39.5	558	29	123.2	4.4	4.4	3.1	3.6	38
10	276.2	27.3	26.5	26.7	26.7	637	Aug. 5	165.5	11.6	6.2	5.0	6.6	95
13	276.3	117.3	22.1	21.9	31.5	752	12	212.8	12.2	10.3	9.0	9.9	183
17	251.5	134.0	—	22.3	34.0	739	19	198.3	5.6	5.6	4.6	5.0	86
20	232.7	89.7	9.1	8.4	24.8	499	26	155.0	4.9	4.8	4.7	4.8	64
24	206.9	10.3	7.8	7.3	8.0	143	Sept. 2	124.3	4.6	4.0	3.9	4.1	44
28	185.6	23.9	12.1	6.3	11.0	176	9	96.8	3.8	3.6	3.6	3.6	30
June 3	150.1	6.0	5.2	4.7	5.1	66	16	134.1	13.1	4.7	5.8	7.0	81
10	133.2	4.6	5.9	4.5	4.8	55							

Table 7. The content of suspended inorganic matter at Edsforsen.

Date	Discharge m^3/sec	Content of silt			Load tons/24 hours	Date	Discharge m^3/sec	Content of silt			Load tons/24 hours
		C_B	C_S	C_M				C_B	C_S	C_M	
1954 Apr. 15	58.2	9.8	6.6	7.6	38	1955 Jan. 7	60.0	1.5	1.7	1.6	8
20	55.2	7.3	8.6	8.2	39	13	61.0	1.5	1.0	1.2	6
22	59.7	4.9	4.5	4.6	24	27	59.7	1.1	1.4	1.3	7
26	72.9	6.6	6.7	6.7	42	Feb. 4	63.4	1.2	—	1.2	7
29	66.3	4.8	3.8	4.1	24	10	52.0	3.3	3.2	3.2	14
May 3	83.2	5.2	4.8	4.9	35	17	46.0	3.0	2.7	2.8	11
6	163.5	40.2	36.7	37.8	534	24	43.0	2.2	2.5	2.4	9
10	276.2	32.0	28.1	29.3	699	March 3	41.2	2.5	2.6	2.6	9
13	276.3	31.5	24.0	26.3	628	10	40.6	1.8	1.8	1.8	6
17	251.5	13.4	12.7	12.9	280	17	38.5	2.2	2.2	2.2	7
20	232.7	9.1	8.4	8.6	173	24	38.0	2.1	2.2	2.2	7
24	206.9	7.6	7.2	7.3	130	31	39.2	2.4	2.2	2.3	8
28	185.6	5.6	5.5	5.5	88	Apr. 7	35.0	—	2.3	2.3	7
June 3	150.1	4.6	4.3	4.4	57	14	28.0	2.3	—	2.3	6
10	133.2	4.5	4.3	4.4	51	21	58.5	2.8	2.4	2.5	13
17	159.5	3.5	4.2	4.0	55	28	61.1	3.4	3.2	3.3	17
July 1	156.9	5.5	5.1	5.2	71	May 2	97.0	14.2	13.4	13.6	114
8	94.5	4.1	3.1	3.4	28	5	157.9	18.6	17.4	17.8	243
15	95.6	3.7	3.7	3.7	31	9	195.5	25.2	24.0	24.4	412
22	160.1	5.2	4.0	4.4	61	12	276.0	28.5	25.6	26.5	632
29	123.2	5.3	5.2	5.2	55	16	190.0	11.9	11.4	11.6	190
Aug. 5	165.5	6.5	4.8	5.3	76	20	238.0	56.5	47.5	50.2	1032
12	212.8	7.5	7.7	7.6	140	23	272.0	25.8	20.7	22.2	713
19	198.3	4.8	4.5	4.6	79	24	309.0	10.4	9.9	10.1	270
26	155.0	4.7	3.5	3.9	52	26	220.8	9.5	7.3	8.0	153
Sept. 2	124.3	4.4	2.7	3.2	34	30	253.0	17.3	12.4	13.9	304
9	96.8	3.1	2.7	2.8	23	June 2	269.0	18.6	16.5	17.1	397
16	134.1	3.7	3.3	3.4	39	6	280.0	21.9	12.4	15.3	370
23	145.9	5.1	4.5	4.7	59	9	295.0	14.6	12.1	12.9	329
30	191.6	3.7	3.2	3.4	56	13	217.1	6.4	5.0	5.4	101
Oct. 7	122.7	3.8	2.7	3.0	32	16	195.5	10.3	8.1	8.8	149
21	84.0	3.0	4.8	4.3	31	21	182.3	5.6	4.1	4.6	72
28	251.5	10.0	7.7	8.4	183	24	177.9	5.2	4.4	4.6	71
Nov. 4	175.0	2.9	2.7	2.8	42	27	168.4	5.8	3.9	4.5	65
11	112.2	2.0	2.0	2.0	19	30	162.3	5.9	3.8	4.4	62
18	53.4	1.4	1.6	1.5	7	July 4	150.6	7.4	4.3	5.2	68
25	70.6	2.1	1.9	2.0	12	7	179.8	7.1	5.7	6.1	95
Dec. 2	116.5	3.1	2.7	2.8	28	11	135.0	4.1	4.4	4.0	47
6	190.4	33.5	30.6	31.5	518	14	139.0	4.8	3.9	4.2	50
9	102.0	6.3	3.5	4.3	38	18	109.3	4.7	3.7	4.0	38
16	133.5	9.0	5.5	6.6	76	21	90.2	3.7	2.7	3.0	23
23	59.8	2.9	1.6	2.0	10	25	105.0	3.3	3.6	3.5	32
30	65.0	2.1	1.6	1.8	10	Aug. 1	68.0	—	3.2	3.2	19
						4	62.0	—	2.4	2.4	13

The curve in the figure is of regular curvature, and would certainly have been closed if the water level had fallen to the value it had before the spring flood. The ice began to break up about the 15th April. The silt content was then c. 8 mg/l, which implies an increase of 2—3 mg/l since before the break-up of the ice. When the rise in the silt content due to the ice break-up ceased, the silt content fell again, and did not rise until the start of the spring flood proper during the first days of May. *The silt content then rose very rapidly,* *and reached a maximum some days before the culmination of the spring flood.* By the time the spring flood had reached its highest level, between the 10th and 13th May, the silt content had already begun to fall.

The relation between the silt content and volume of flow illustrated by the curve is typical of heavy floods (cf. the comprehensive list of references in HJULSTRÖM 1935, p. 410), although the present curve is exceptionally regular. The regularity of the curve is probably due to the fact that the melting of the snow and the run-off was but little disturbed by rain in the spring of 1954.

The loop shape of the curve may be accounted for as follows. As the river level rises, the water in the main stream and in tributary streams gradually reaches surfaces where there are loose particles that are easily swept away. Interfluvial erosion also plays an important part. The most easily entrained particles are soon carried away, however, and the material of the river bed partially adjusts itself to the new flow conditions. The erosion then diminishes, and the silt concentration falls, although the volume of flow has not reached its maximum.[1]

The variation of the sediment load is similar whenever there is heavy rain. But there may be a temporary increase in the silt concentration even on other occasions. Sometimes a slide on some terrace may lead to a large rise in the silt load: this occurred on the occasion of the latest slide on the terrace at Ändenäs in 1951, according to local information. The silt content is also high when temporary ice forms or breaks up during the autumn or early winter, even though the volume of flow may not be particularly high (cf. the figures from Värnäs for the 28th Nov. 1953, and from Edsforsen for the 6th Dec. 1954).

In view of the above-mentioned deviations from a single-valued relation between the volume of flow and the silt content, and of the relatively short period during which measurements were made (the spring flood was unusually small during 1954, 1955, and 1956), it was not considered possible to construct a *silt-discharge rating curve.*

The total amount of suspended material passing Edsforsen may be seen from the diagram in fig. 59, where the relation between the discharge and the transportation of bed load is also illustrated. The greatest load of sediment was noted on the 20th May 1955, 1,032 tons during 24 hours. The discharge on this occasion was 238 m^3/sec, and at the culmination of the spring flood some days later 309 m^3/sec. The discharge at the culmination of a normal spring flood is c. 650 m^3/sec, so it is easily seen that the silt load should be considerably greater during a normal spring flood. The maximum is probably 2,000—3,000 tons/day in normal years, and on exceptional occasions considerably higher.

The period during which measurements were made was far too short to enable any statement to be made about the normal annual transportation of sediment, especially when the unusually small spring floods are taken into consideration. During the 12 month period from May 1954 to April 1955 the total amount of sediment passing Edsforsen was

[1] HJULSTRÖM (1935, p. 411) has found that for the river Fyris "the direct erosion of the river . . . is, therefore, of less importance than the inter-fluvial erosion by weathering processes and rain-wash". However, the geological conditions there are quite different, and it is not surprising that the suspended material in the river Klarälven to a much greater extent is taken up on erosion of the stream bed.

about 25,000 tons, and during the period from August 1954 to July 1955 about 32,000 tons. The normal value is probably much higher.

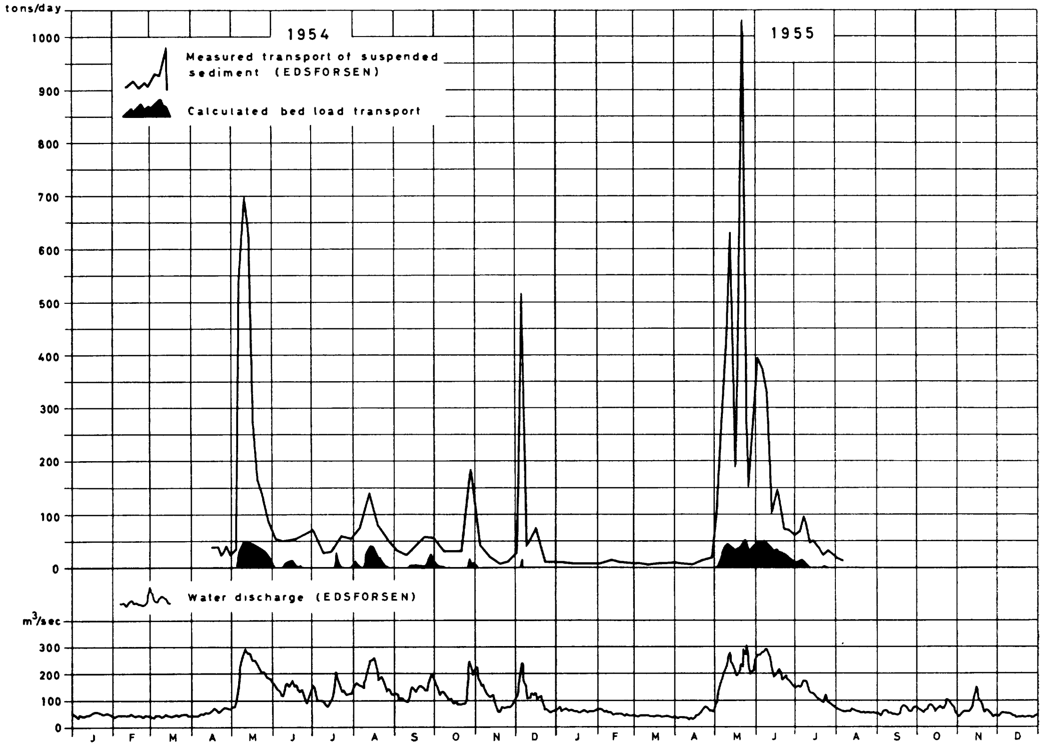

Fig. 59. The sediment load at Edsforsen during the period April 1954—Aug. 1956.

The grain-size distribution of the suspended material

The grain-size distribution of the suspended material is intimately connected with the flow velocity and the state of turbulence. In general there is a maximum grain size for each velocity of flow (cf. fig. 23). The grain-size distribution determines the vertical distribution of sediment in the stream channel (cf. eq. 56).

It is therefore a matter of considerable interest to study the distribution of grain sizes at different levels in the river, and under different flow conditions. On account of the low content of sediment in Klarälven, very large samples of water are required for an ordinary sedimentation analysis. The present investigation has not included any such analyses. But in some cases sediment has been analysed microscopically.

The analyses were carried out by counting the number of particles in various intervals of grain size, after which the numbers of particles were transformed to percentages by weight. The ignition residue from a filter paper was mixed with glycerine, and particles then counted with the help of an eyepiece graticule (cf. ANDREASEN 1939, p. 13).

The results of some of these analyses may be seen from the table below.

Table 8. The distribution of grain sizes in suspended sediment (in percent by weight).

Date	Locality	Level	0.6—0.2 mm	0.2—0.06 mm	0.06—0.02 mm	0.02—0.006 mm	< 0.006 mm
1953. Oct. 3........	Värnäs	B	10	22	31	25	12
»	Värnäs	S	0	18	37	28	17
1953. Oct. 24.......	Värnäs	B	2	19	32	28	19

Bed load

In connection with an investigation regarding the effect of proposed regulations of the run-off on erosion in the meander zone of Klarälven, the author has calculated the probable amount of material transported as bed load north of Edebäck at different volumes of flow (SUNDBORG 1956 b).

The calculations, which made use of KALINSKE's formula for bed load (cf. p. 213) need not be described in detail here. They were divided into two parts: firstly, a determination of the critical volume of flow at which a general erosion of the river bed commences, and secondly, a determination of the relationship between the bed load and the volume of flow for values of the latter exceeding the critical value.

Since conditions may vary widely from one transverse section to another, 4 different sections north of Edebäck were selected for calculations. In order from the south they were at Ängsåsen, Nästorp, Ämtbjörk, and Bergsäng (cf. Pl. I).

Soundings were made in each section, and samples of the bed material taken (fig. 50). A discharge rating curve was also drawn up for each section, and the average flow velocity for the transverse sections was determined as a function of the volume of flow.

The result of the determination of the critical volume of flow was that general transportation of bed load commences in the section at Ängsåsen when the volume of flow is c. 250 m^3/sec, at Nästorp at c. 200 m^3/sec, at Ämtbjörk at c. 160 m^3/sec, and at Bergsäng at c. 250 m^3/sec, when the dammed water stage is at its winter value (135.50 m.a.s. at Edebäck). At the summer water stage (135.00 m.a.s. at Edebäck), general transportation of bed load commences at Ängsåsen at c. 170 m^3/sec, at Nästorp at c. 140 m^3/sec, at Ämtbjörk at c. 110 m^3/sec, and at Bergsäng at c. 180 m^3/sec.

The calculations of the relation between bed load and water discharge showed that there are considerable differences between the four sections. This is to be expected in view of the fact that a river bed composed of loose sediment is not stable. When the flow of water slackens there is deposition of material, and the section becomes shallower; when the flow is more rapid the river channel is deepened if the material arriving from upstream is less in amount than that removed. With the present large variation in Klarälven's volume of flow there is never time for the river bed to attain a state of equilibrium where the transportation of bed load is the same for the whole meander zone. Instead, erosion and transport are concentrated to certain parts of the river, while the river is elsewhere less active. In a particular section a deepening of the channel entails an increase in the cross-sectional area, and consequently a decrease in the action of the water on the river bed, that is, a decrease in the transportation of bed load. Conversely, if the river becomes shallower the transportation of bed load usually increases, if the volume of flow remains the same.

The average figures for the transportation of bed load in the four sections indicate that when the river is at its regulated summer stage bed load is insignificant for volumes of flow less than 150 m^3/sec and when the river is at its winter stage for less than 200 m^3/sec. When the flow rises above these values the bed load increases more rapidly than the volume of flow, i.e. the relation between them is not linear.

In fig. 59 the bed load has been computed with the aid of the calculated relation between volume of flow and bed load. According to the calculations the bed load is usually small in comparison with the load of suspended material, being about 10—15 % of the total load during the period of the investigation. However, these figures must be taken with reservation, since the methods used for calculation of the bed load are rather imperfect. The error in the calculated values is probably some 50—75 %.

But the general trend of the curve relating bed load and volume of flow is undoubtedly correct. An observation confirming this is that at volumes of flow for which the calculations indicated that the bed load should be appreciable it was found that the silt content in the water samples from near the river bed at Sälje contained a relatively large amount of silt compared with the samples from higher levels, which indicates that the sand on the river bed had been set in motion.

SOME VIEWS ON THE POSTGLACIAL EVOLUTION OF THE VALLEY OF KLARÄLVEN

De Geer's view was that the terraces along the sides of the present valley were formed during the postglacial period by the river as it cut out its present valley. von Post, on the other hand, has interpreted the terraces as ancient marine shore terraces (cf. p. 241). These sharply different views concern a central problem in the history of the river, and it is appropriate to consider this problem in the light of the fluvial processes and fluvial morphology as they have been presented here.

von Post's article (1948) is "a preliminary report about a research on the raised beaches of the ancient fiord that filled, in Late-Glacial and early Post-Glacial time, the rock valley stretching from Norra Finnskoga, in the northernmost part of Värmland, to Brattforsheden, about 170 km southwards, part of which is now occupied by River Klarälven".

von Post (pp. 197—198) states, in opposition to De Geer's view, that "it was from the beginning fairly clear to me, however, that these terrace steps must in the main be ancient marine beaches. For, formations corresponding to the terraces of the river valley are to be found in environments where river erosion cannot have contributed to their creation".

Since von Post considered that he could connect shore terraces in neighbouring valleys where there is no river with the terraces in the valley of Klarälven, he was obliged to assume that there was once an extensive horizontal water surface in the valley of Klarälven as well. "In the part of the ancient fjord which is now the river valley the water level was determined by the base-level at sea-level until the time when the threshold of the pass at

Edebäck rose above sea-level and became the base-level for the river. Up to that time the valley north of Edebäck was an estuary, where, although the water was flowing, there was practically no surface gradient. Not until afterwards did there develop a fall gradient on the slowly moving part of the river between Sysslebäck and Edebäck, where the mean gradient is now 0.13 ‰" (p. 200).

These assumptions made by VON POST are of course absolutely necessary for his interpretation of the origin of the terraces, in view of the fact that the abrasion must have been sufficently strong to have cut out the terraces, and in view of the supposed conformity with the gradients of the shore terraces in neighbouring valleys. It may be questioned whether the stratigraphy of the ancient fjord sediments supports VON POST's hypothesis of an estuary in the upper valley of Klarälven.

The sediments of which the valley terraces are composed are mostly sand and silt. The surface of the terraces is generally much flatter than that of the present meander headlands, but often slightly undulating on account of what appears to be stream furrows. The slope of the surface outward from the valley side to the edge of the terrace is slight but often distinct, especially for the highest terrace. Where the author has been able to examine the stratification of the sediment (which is visible in a large number of places, e.g. in ravines that cut into the terraces), it has always been very regular, distinctly varved, and with only insignificant disturbances of the primary stratigraphy.

The deposition of varved sediment along the valley sides while the central part of the valley was supposed to remain free of sediment has been explained by VON POST (p. 205) as due to the presence of an ice remnant in the valley. "An ice remnant—a glacial lobe that was climatically dead but to begin with dynamically active—filled the valley from Brattforsheden to Norra Finnskoga. The existence of such a limb of ice is confirmed by the marginal channels mentioned above. These occur at practically all levels of the lower parts of the valley sides, but cease at MG."

Although it is likely that a lobe of ice stretched southward along the valley at an early stage of the deglaciation, it is difficult to see how it could have persisted more or less undisturbed in the relatively deep fjord during the final stage for a period sufficiently long for *undisturbed deposition* of well-sorted sediment along the valley sides. The undisturbed stratification of the thick deposits undoubtedly required more stable sedimentation conditions than those associated with an ice remnant that was shrinking and breaking up. The melting of an ice remnant would have been accompanied by the caving in of sedimentary deposits along the sides, and very great disturbance of the primary stratification. The only part of the meander zone where the development may possibly have followed these lines is the region around Vingängsjön and northward.[1]

The distinct unconformity between the underlying fjord sediment and the overlying river sediment (cf. p. 273) also indicates that the valley must have been filled with sediment

[1] Vingängsjön is situated in a depression in the loose sedimentary deposits, a depression that was probably occupied by an ice remnant during the period of deglaciation. The depression has subsequently been partially filled with younger sediment. Vingängsjön has gradually shrunk, partly on account of a delta that has encroached on the lake, and partly because the river below the lake has gradually cut a deeper channel in the sediment there, thereby slowly draining the lake.

to a level higher than would permit VON POST's estuary to have existed. At Månäs in Norra Ny parish (see Pl. I) a terrace projects out into the middle of the valley, and a small erosion remnant on the other side of the valley extends almost to meet it. Embedded in the fine-grained sediment of the western terrace are large boulders, the sequence of strata being otherwise undisturbed. These boulders must have been carried by floating icebergs, which means that the deposits were certainly laid down during the period of deglaciation.

On the terrace at Ändenäs in Norra Ny parish (p. 265) the lowest varves of the terrace sediment can be followed eastward below the surface of unconformity and the river sediments of the meander headland. This is a convincing proof that overlying varves of the fjord sediment must have been cut away by the river in the central part of the valley.

Consequently, the idea that there was once an estuary with a broad free surface of water in the upper part of what is now the valley of Klarälven during the whole period under which the succession of terraces developed must therefore be rejected. VON POST's view that most of the terraces are marine shore terraces falls with this idea.

VON POST's interpretation of the terraces having been rejected, it remains to be discussed whether the interpretation of the terraces as in the main fluvial erosion terraces can be correct. The observations already put forward indicate that it may be. There are also some additional facts that support DE GEER's view.

The surfaces of the lower terraces often show traces of shallow furrows, which must be regarded as originally stream channels. The surface is seldom or never so regularly undulatory as that of the present meander headlands, and there are no point bars in the true sense on the terraces. Where a lower terrace borders on a higher terrace the boundary between them is at times a roughly circular arc. This shape indicates that a river channel has cut into the overlying deposits and formed a curved erosion scarp. An example of such a formation is to be found on the eastern side of the valley roughly opposite the tip of Ändenäs (not visible in Pl. I, but discernible in fig. 56).

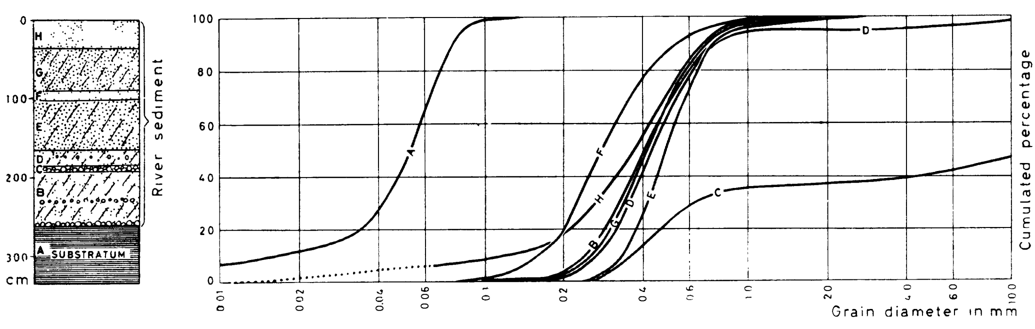

Fig. 60. Stratigraphic section in the upper part of the 28 m-terrace at Ändenäs. Further explanation in the text.

However, the most important evidence of the terraces' fluvial origin is the stratigraphy of the sediment. Fig. 60 shows a section cut into the terrace at Ändenäs. At the bottom of the profile is the varved fjord sediment (analysis A in the diagram on the right). There is a surface of unconformity between this sediment and an overlying stratum of coarser material. At the surface of unconformity is a thin layer of gravel and stones, which may be described as an erosion pavement. The sediment above the surface of unconformity is distinctly cross-laminated, with the dip directed in the main down the valley (analysis B). There are thin gravel laminae at the lower levels.

About 70 cm above the surface of unconformity there is a further layer of gravel with a bimodal grain-size distribution (analysis C; cf. also fig. 15). Like the gravel laminae already referred to, this layer is residual material, and may be regarded as a temporary erosion pavement.

The grain size diminishes upward on the whole, although there is some alternation of finer and coarser material (analyses D, E, F, G, and H). The strata are usually cross-laminated, especially those lower down, and the material must have been mainly transported as bed load. In the upper part of the section it seems that some suspended material occurs (analysis H). It is also possible that the relatively high proportion of finer material there may be due to eolian material.

The thickness of the whole layer above the surface of unconformity is 260 cm in the section, and the terrace is c. 28 m above the average level of the river. The surface of unconformity slopes slightly towards the valley side, and the layer therefore becomes thicker in that direction. The surface of unconformity is characteristic of one cut out by a stream where it is diverted by the valley side.

The stratigraphy described above is very similar to that found in the erosion scarps of the meander headlands (cf. fig. 47). But an important difference is that material such as the point bars are composed of does not occur in the Ändenäs terrace. There can hardly be any doubt that the upper part of the terrace is composed of fluvial sediment, mainly transported as bed load. It was deposited in a water-course which had earlier cut away the upper parts of the fjord sediment. A similar stratigraphy has been observed at several other places in the valley, among them the large terrace at Månäs.

Hence both the morphology and the stratigraphy indicate that the terraces investigated are of fluvial origin. At the *earliest* stages in the river's down-cutting, however, local bulges of the river may have had sufficiently large water surfaces for some abrasion terraces to develop, although the present writer has not been able to trace any. It should be noted that the opinion stated does not mean that terraces in the valley cannot be correlated as synchronous. But surfaces derived as fluvial terraces will undoubtedly have a certain slope, corresponding to the gradient of the river that generated them. It is of course also probable that the manner in which the uplift of the land proceeded resulted in terraces at certain more persistent levels. It is also possible that transgressions may be traceable in the positions of the terrace surfaces. But these possibilities have not been examined.

FUTURE DEVELOPMENTS

In view of the fact that the continuous displacement of the river channel in the meander zone is associated with a down-cutting, which has been quite considerable in the upper part of the zone, the river there has acquired an incised meander course. The down-cutting may be expected to diminish in the future, so that the lateral erosion will in time dominate completely.

De Geer (1911, pp. 165—166) has on this basis forecast the future development of the river: "When the river's down-cutting has practically ceased, the sediment deposits in the river can never project above the high-water stage of the river. The higher, older parts of the meander headlands will be demolished during less than a quarter of a serpentine period, after which no point of the whole valley floor can rise above the high-water level of the river.

All buildings must then be moved close to the sides of the valley ... Farming will have to be modified ... the regions at present liable to flooding are only usable for pasture, while arable land is on the whole above the 5-metre line, i.e. on the central and northern parts of the headlands. Thus the pasture land will increase at the cost of the arable land, until the latter has entirely disappeared ...

If it is considered desirable to attempt to retain the arable land, there are two possible courses to adopt. The building of dams around each headland would be too large an enterprise in relation to the size and value of the acreage involved. But damming of the outlet from Fæmunden would to some extent lessen the risk for flooding and for high river stages by a more even distribution of the flow throughout the year. However, the most effective method would be to put new life into the enfeebled vertical erosion. A canal could be blasted through the rock threshold at Edebäck, whereupon the base-level of the river would sink, and the river would be able to excavate the valley once more, so to speak."

It should be noted that De Geer's forecast presumes that the lateral erosion continues everywhere within the meander zone. The comparative maps of the present publication have clearly shown that the speed of the meander loops' lateral displacement varies greatly in different parts of the meander zone. The shore lines of several meander headlands have remained practically unchanged during the last 150 years.

If the hydrological conditions remain unaltered in the future, there is no reason to suppose that there will be any general intensification of the lateral erosion in places where it has already come to a standstill for some reason (cf. p. 254). Only where the erosion has been temporarily hindered by some impermanent obstacle can renewed activity be expected. Hence there should be very little risk of the above development for headlands that are already stable.

In places where the lateral erosion is comparatively rapid the situation is obviously quite different. It is possible that the river activity in such regions may in time diminish, if the channel reaches more erosion-resistant material. The possibility—or perhaps rather, the probability—that the valley floor will become flatter and more subject to floods in the distant future cannot be excluded. However, flooding leads to sedimentation close to the river. If the lateral erosion does not proceed with exceptional rapidity, the river will be bordered by normal levees, on certain stretches at least. In actual fact the most southerly part of the meander zone in the Ekshärad district already exhibits pronounced levee formations near the river, a consequence of the almost complete cessation of vertical erosion in the region immediately upstream from the local base level.

The formation of levees lessens the risk of repeated floods. Thus there seems to be no danger that a radical alteration of the pattern of habitation and agriculture will be necessary. But it is probable that the really catastrophic floods will in the future lead to inundation on a scale at least as large as at present, unless the natural levees are reinforced by dikes.

These conclusions assume that the evolution of the river is not altered by external measures. Apart from purely local operations such as the building of revetments and dikes, the natural evolution may be influenced in two ways, both of which have been touched upon by De Geer.

The local erosion base at Edsforsen may be altered. An alteration of present interest is a raising of the dammed level to give a better head of water at the power station. This would lead to a raising of the water level in the region above the dam, especially at low water. Better precautions against flooding would then be required. The effect of such a measure on the processes of erosion, transportation, and sedimentation cannot be estimated without a special investigation, but it is likely that there would be some deposition of sediment upstream from Edsforsen.

The other type of measure is a regulation of the run-off by dams at different parts of the river. The purpose of such dams would be to create a more even discharge so that the water power can be utilised more efficiently. Since the ability of the river to erode and transport sediment does not vary linearly with respect to the discharge, the increase being more rapid at high volumes of flow, a smoothing-out of the variation in the river's discharge would lead to a decrease of the total erosion and transportation of sediment during the year. A forecast of the effect of further damming would of course have to consider many subsidiary problems as well, which cannot be discusssed here.

To summarise, it may be said that there is no good reason to suppose that De Geer's pessimistic view of future developments will be realised. Although there will no doubt continue to be a great deal of damage from erosion and flooding, it is likely that the situation will improve to some extent in the future, as regards erosion damage at least.

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Some frequent abbreviations:

Bull. Am. Ass. Petr. Geol. — Bulletin of the American Association of Petroleum Geologists.
Bull. Geol. Soc. Am. — Bulletin of the Geological Society of America.
Civ. Eng. — Civil Engineering.
Geogr. Ann. — Geografiska Annaler.
G. F. F. — Geologiska Föreningens i Stockholm förhandlingar.
Jour. Sed. Petr. — Journal of Sedimentary Petrology.
Medd. Stat. Met. Hydr. Anst. — Meddelanden från Statens Meteorologisk-Hydrografiska Anstalt.
Medd. Sv. Met. Hydr. Inst. — Meddelanden från Sveriges Meteorologiska och Hydrologiska Institut.
Phil. Mag. — Philosophical Magazine.
Phil. Trans. Roy. Soc. — Royal Society of London, Philosophical Transactions.
Proc. (Trans.) Am. Soc. Civ. Eng. — Proceedings (Transactions) American Society of Civil Engineers.
Proc. Roy. Soc. — Proceedings of the Royal Society of London.
S. G. U. — Sveriges Geologiska Undersökning.
Trans. Am. Geoph. Union — Transactions American Geophysical Union.
Univ. Iowa Stud. Eng. — University of Iowa, Studies in Engineering.
Zeits. ang. Math. Mech. — Zeitschrift für angewandte Mathematik und Mechanik.

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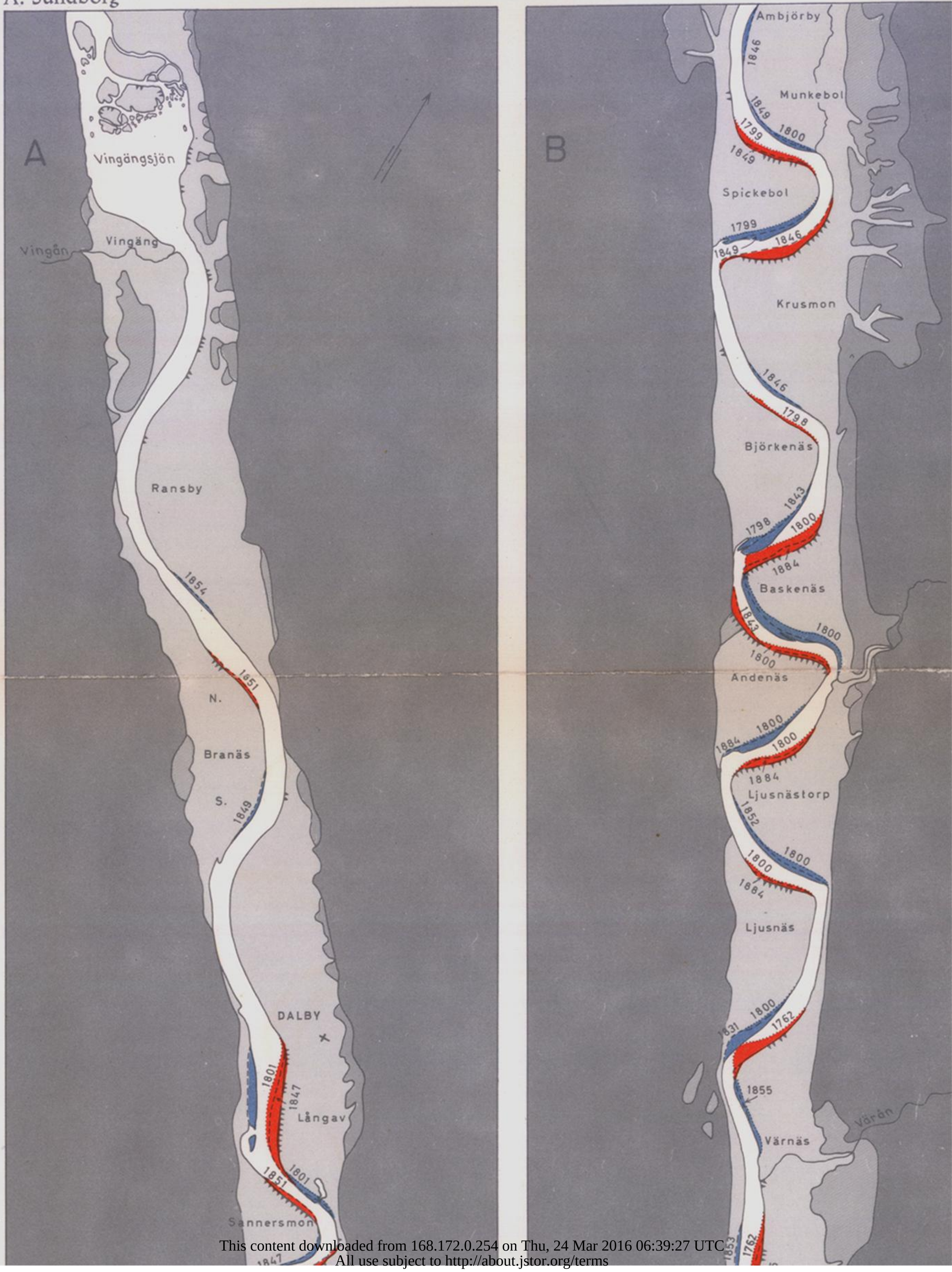
KLARÄLVENS ME

(THE KLARÄLVEN M

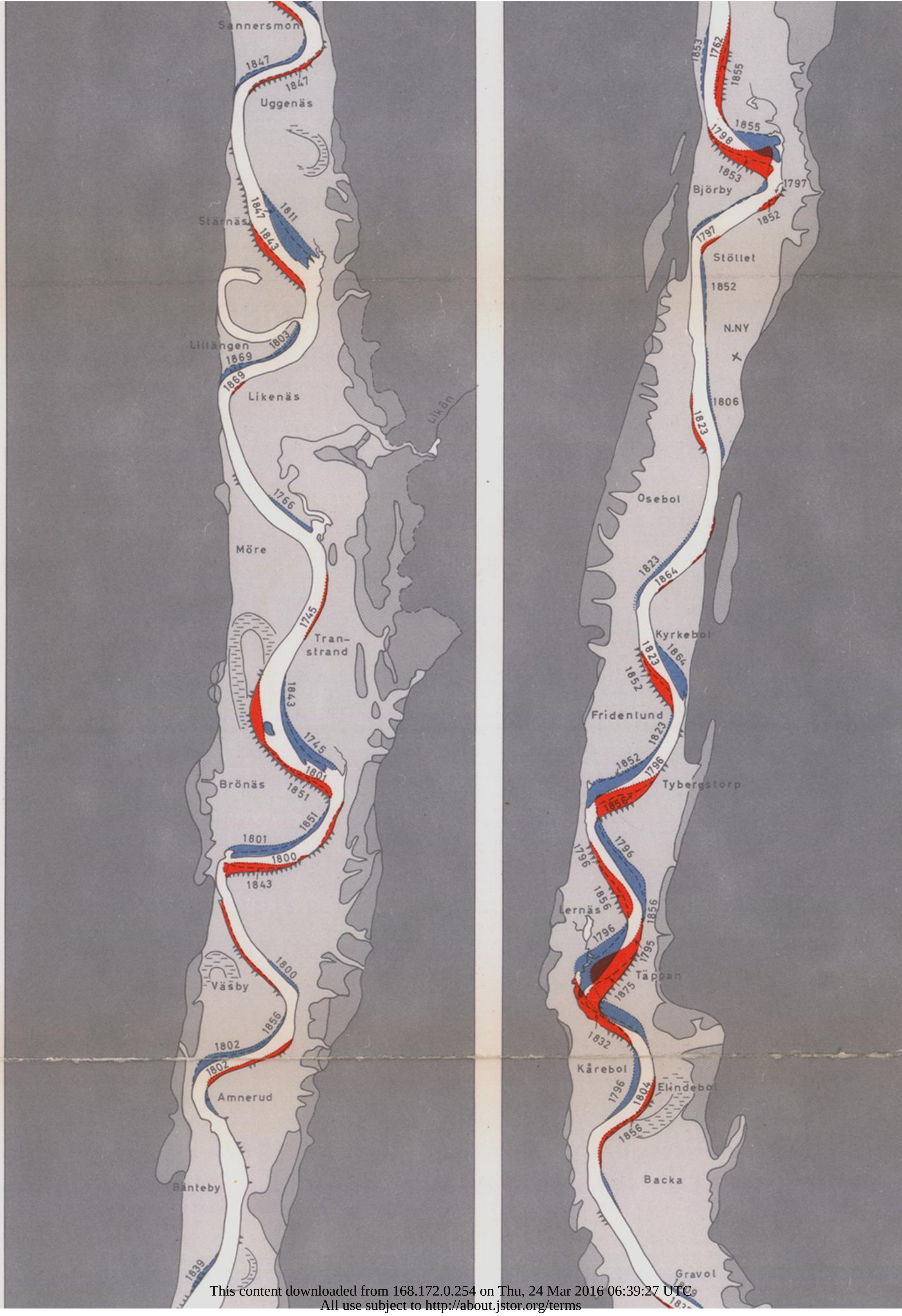
Å. Sundborg

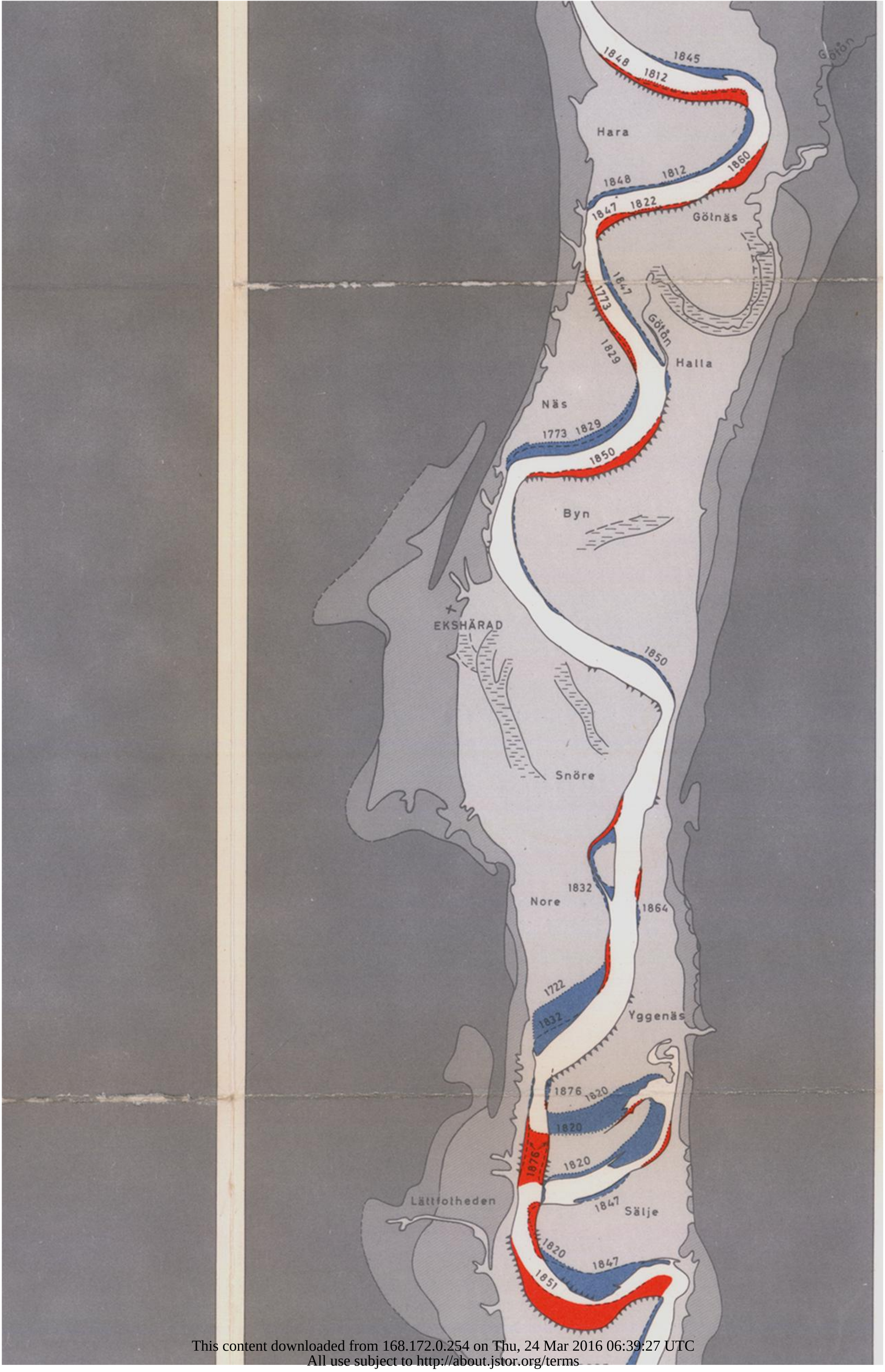


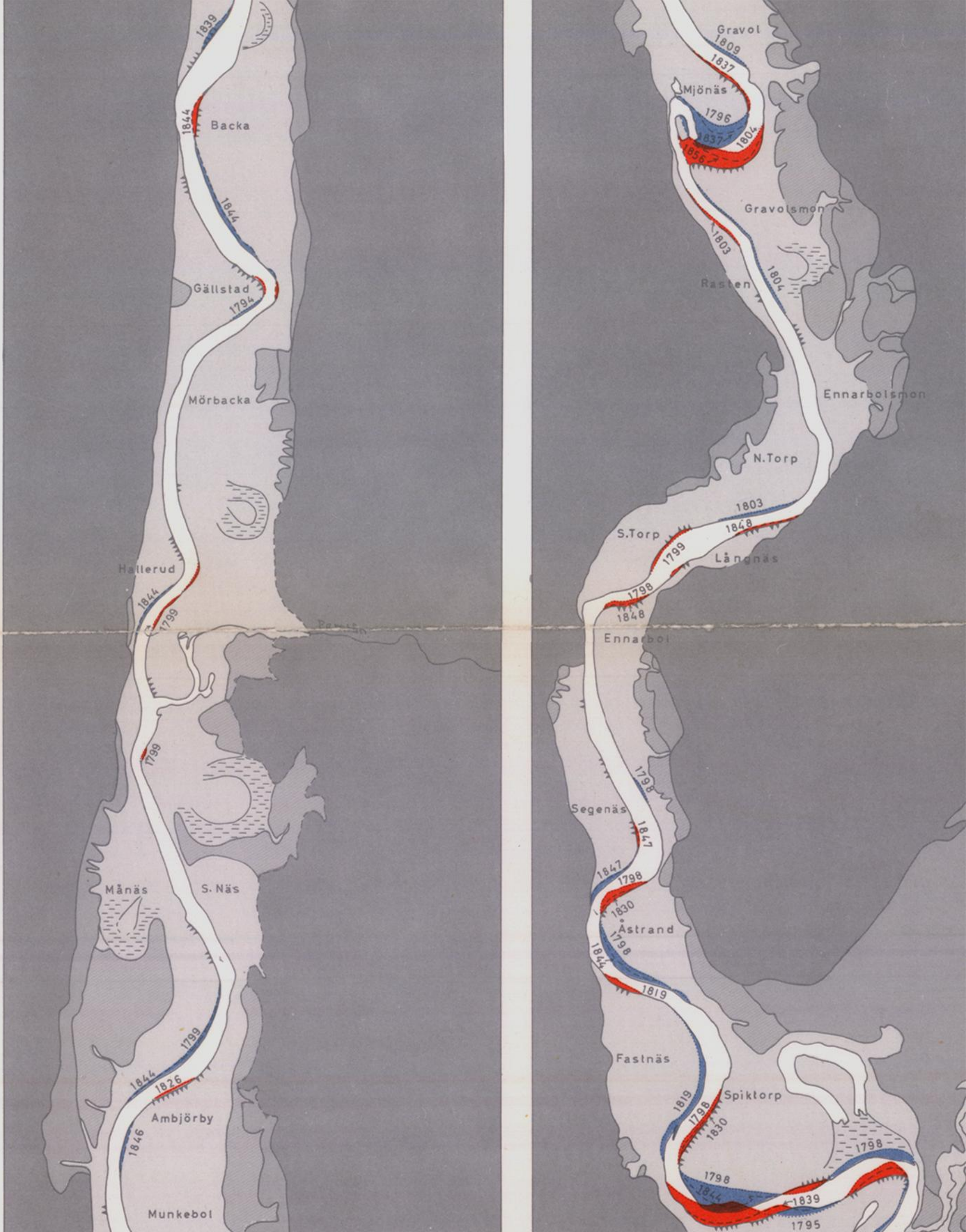
MEANDERLOPP

EN MEANDERS)

Geografiska Annaler 1956, Pl. I


Sannersmon
Uggenäs
Stärnäs
Lillängen
Likenäs
Likån
Möre
Tran-
strand
Brönäs
Väsby
Amnerud
Bänteby
Björby
Stöllet
N.NY
Osebol
Kyrkebol
Fridenlund
Tybergstorp
Lernäs
Täppan
Kårebol
Elindebol
Backa
Gravol


Götån
1848
1845
1812
Hara
1848
1812
1860
1847
1822
Götnäs
1847
1773
Götån
1829
Halla
Näs
1773
1829
1850
Byn
EKSHÄRAD
1850
Snöre
1832
Nore
1864
1722
1832
Yggenäs
1876
1820
1820
1876
1820
Lättfotheden
1847
Sälje
1820
1847
1851



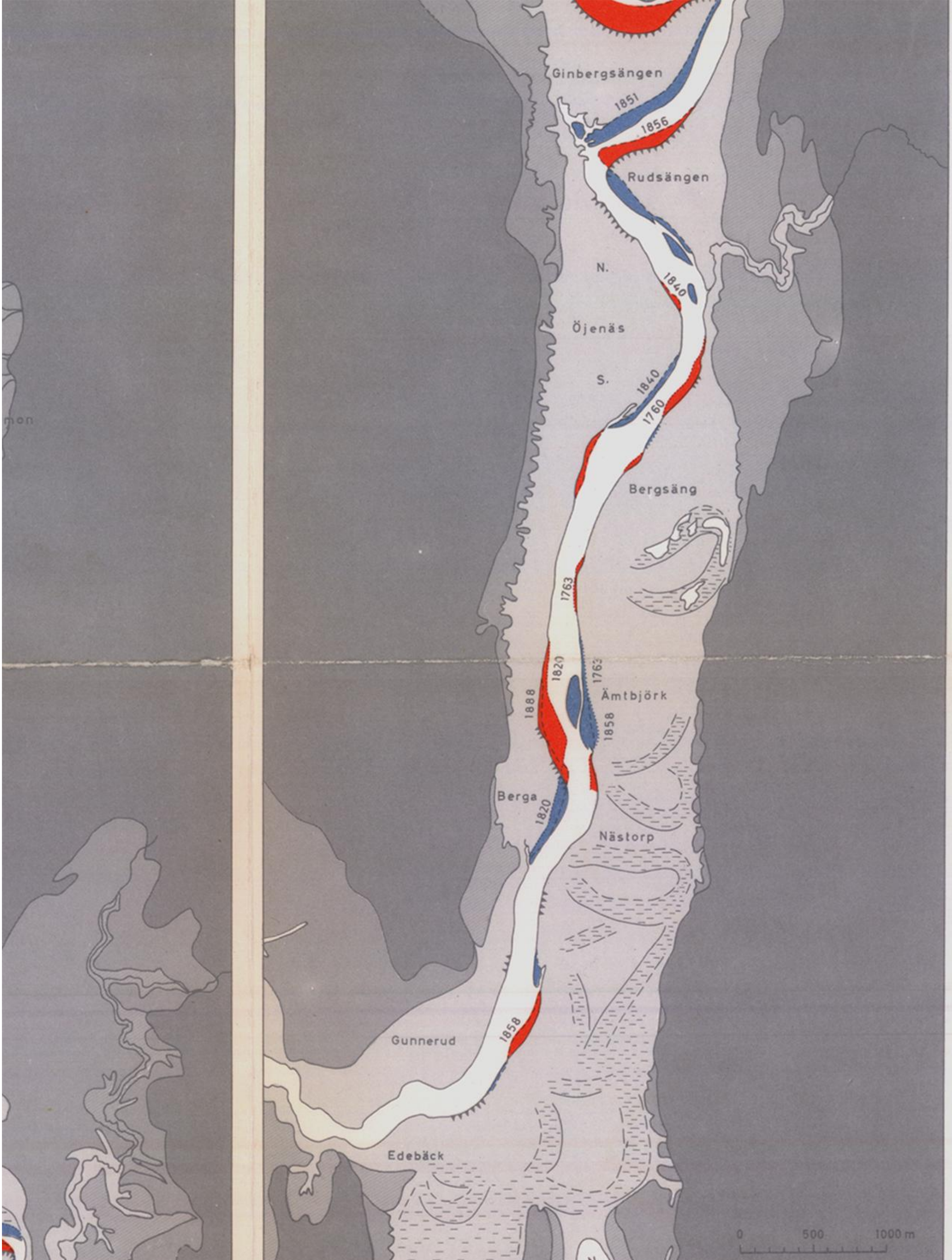
FÖR PUBLICERING GODKÄND I RIKETS ALLMÄNNA KARTVERK DEN 9/2 1956

1806
1851
Strandkonturer
(Banklines)

Borteroderat område
(Bank recession)

Ackumulationsområde
(Bar building)

Aktiv erosion 1954



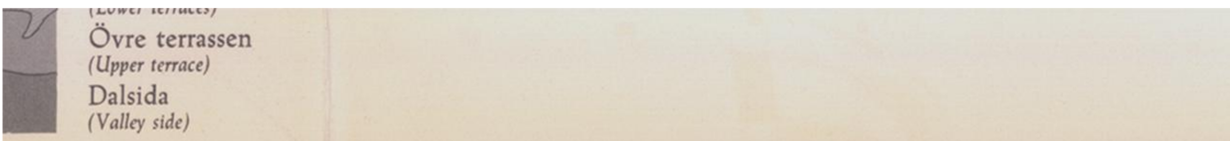
TRYCKT VID GENERALSTABENS LITOGRAFISKA ANSTALT STOCKHOLM 1956

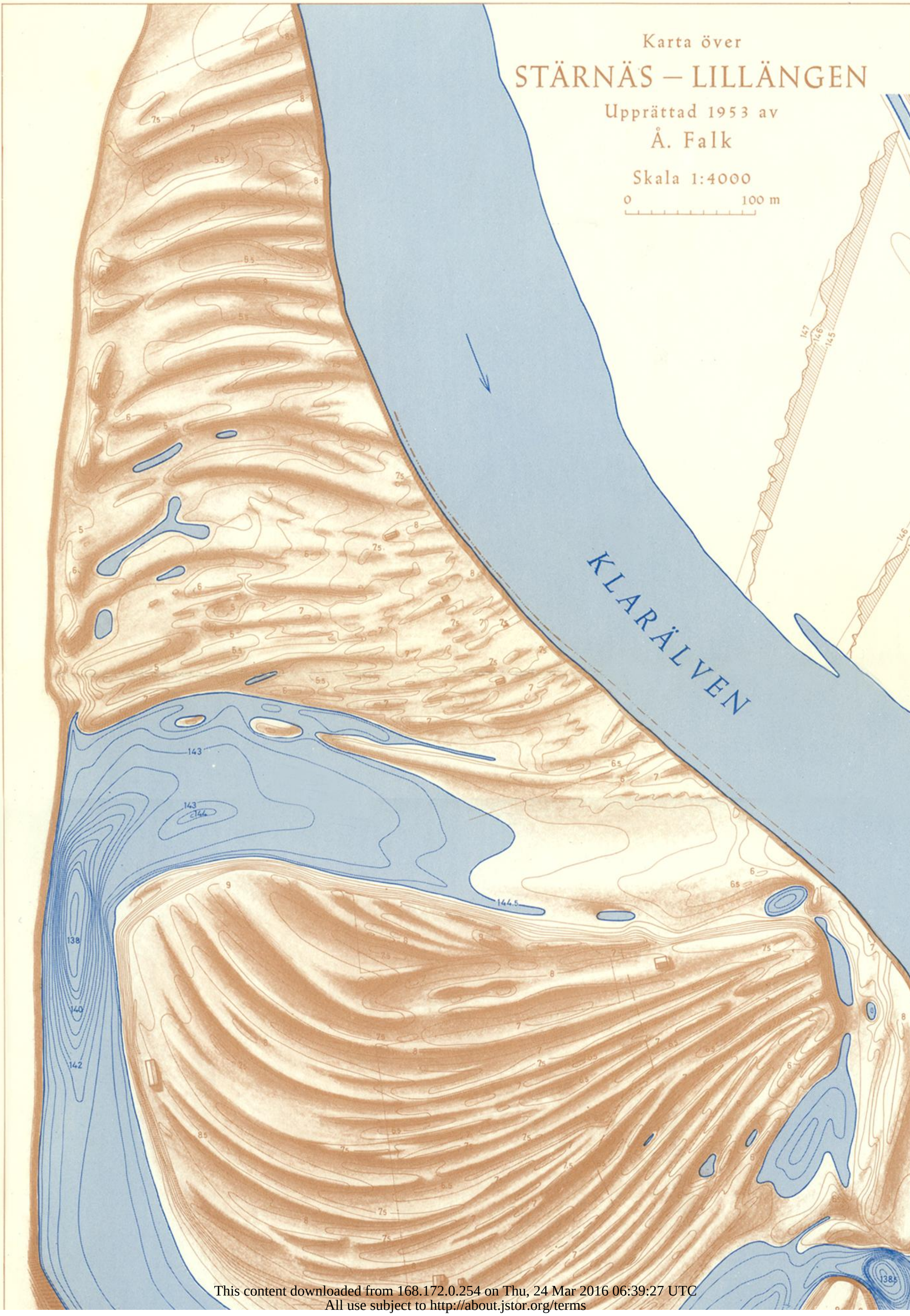
Dalbotten med övergiven meanderbåge
(Valley floor with abandoned loop)
Undre terrasser
(Lower terraces)
Övre terrassen
(Upper terrace)
Dalsida

(Bank recession)

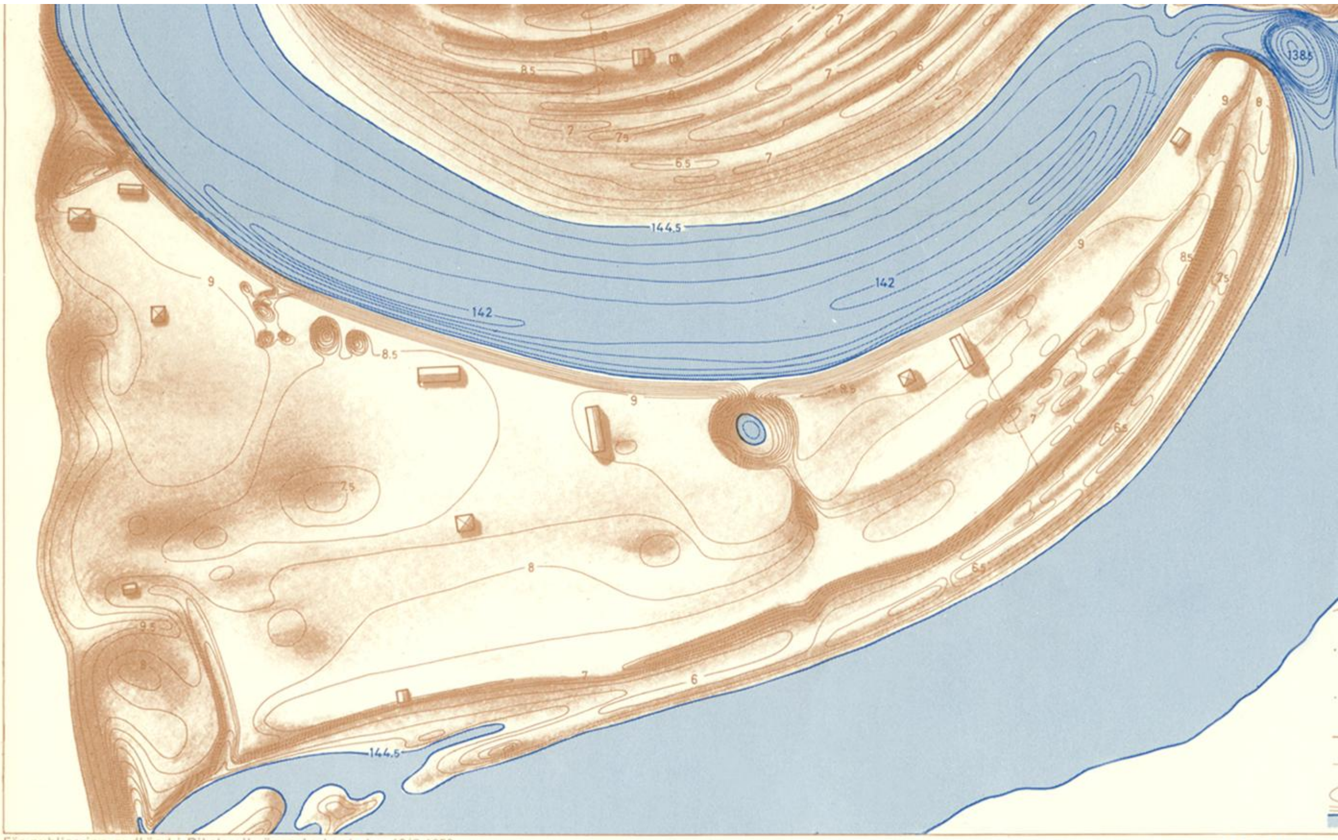
Ackumulationsområde
(Bar building)

Aktiv erosion 1954
(Erosive activity 1954)


Övre terrassen
(Upper terrace)
Dalsida
(Valley side)

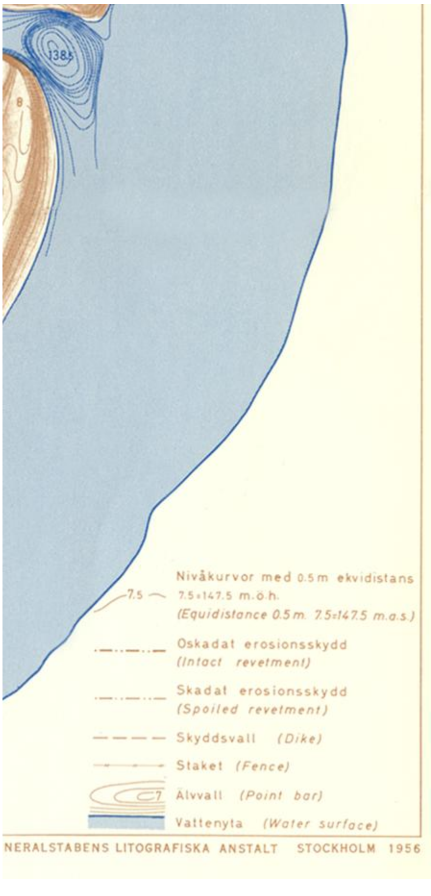

Karta över
STÄRNÄS – LILLÄNGEN
Upprättad 1953 av
Å. Falk
Skala 1:4000
0
100 m
KLARÄLVEN
143
143
144
144.5
138
140
142


N
5.5
10
14.6
14.5
7
8
1385



För publicering godkänd i Rikets allmänna kartverk den 18/7 1956
Av Kartverkskommissionen godkänd för spridning den 20/7 1956

GENERALSTABEN



138.5
8
7.5
Nivåkurvor med 0.5 m ekvidistans
7.5=147.5 m.ö.h.
(Equidistance 0.5 m. 7.5=147.5 m.a.s.)
Oskadat erosionsskydd
(Intact revetment)
Skadat erosionsskydd
(Spoiled revetment)
Skyddsvall (Dike)
Staket (Fence)
Älvvall (Point bar)
Vattenyta (Water surface)
NERALSTABENS LITOGRAFISKA ANSTALT STOCKHOLM 1956